Anisotropic poroelasticity of rocks and its effective stress dependency

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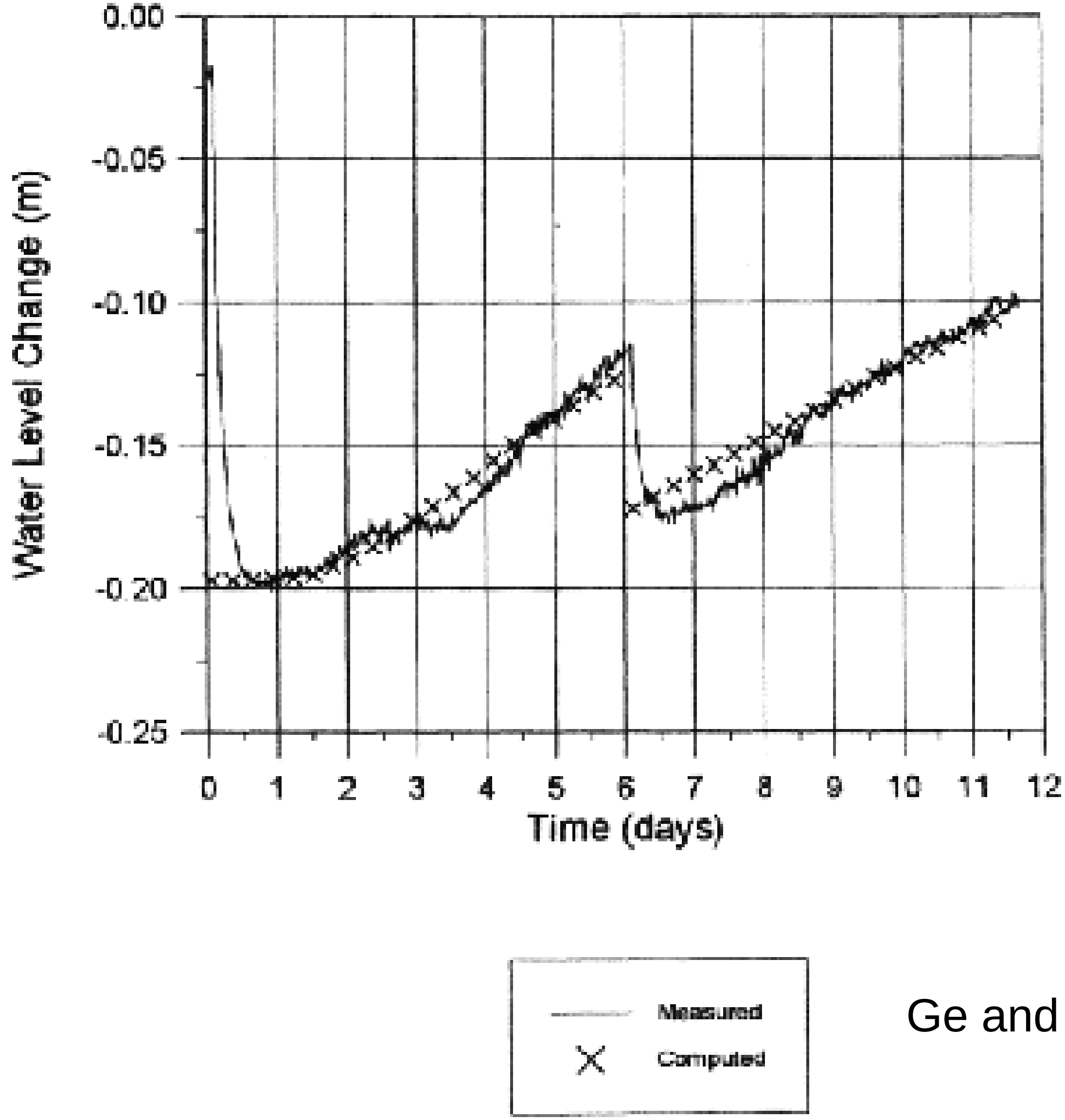
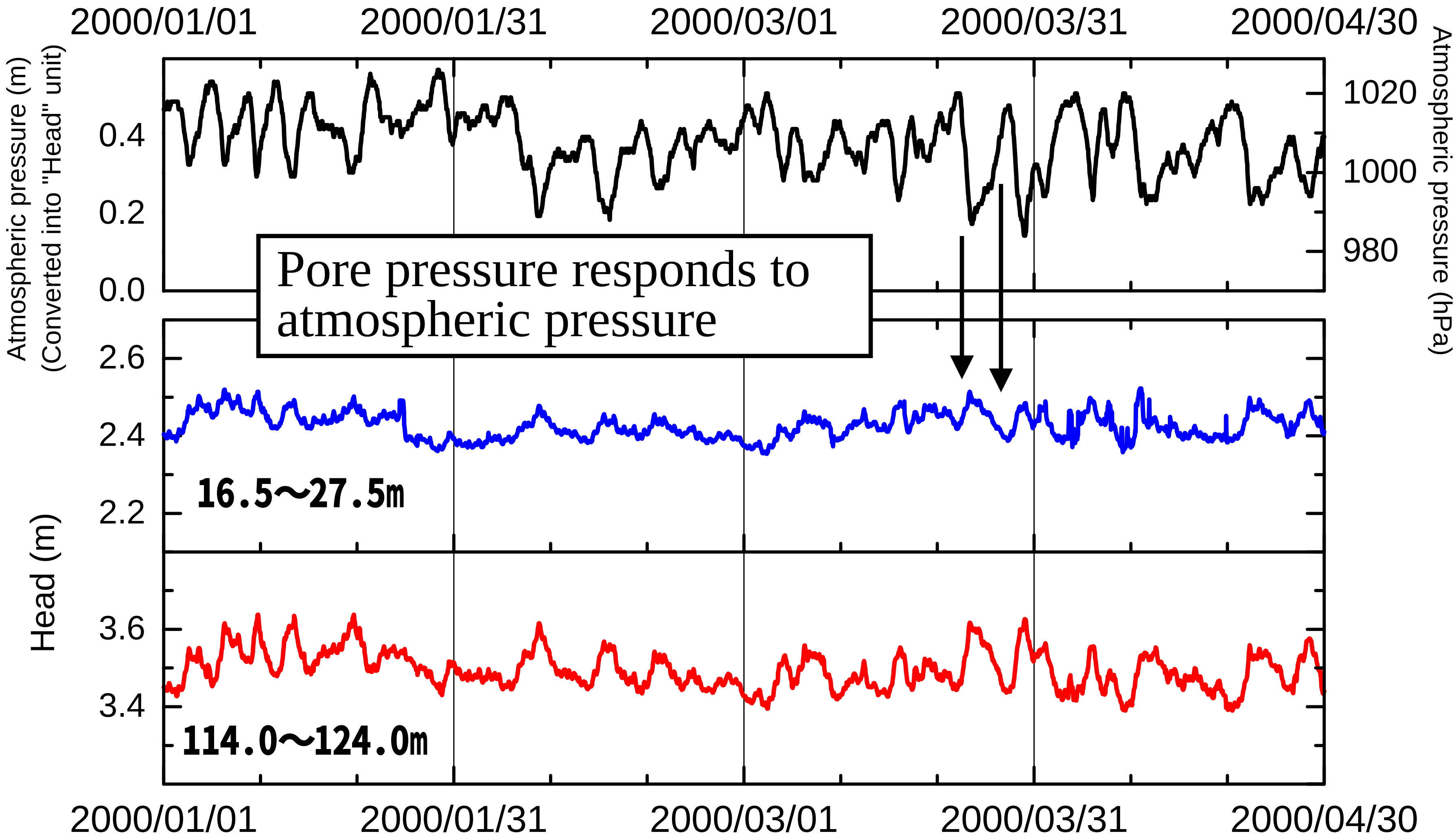
Example of earthquake-related water level change in Parkfield

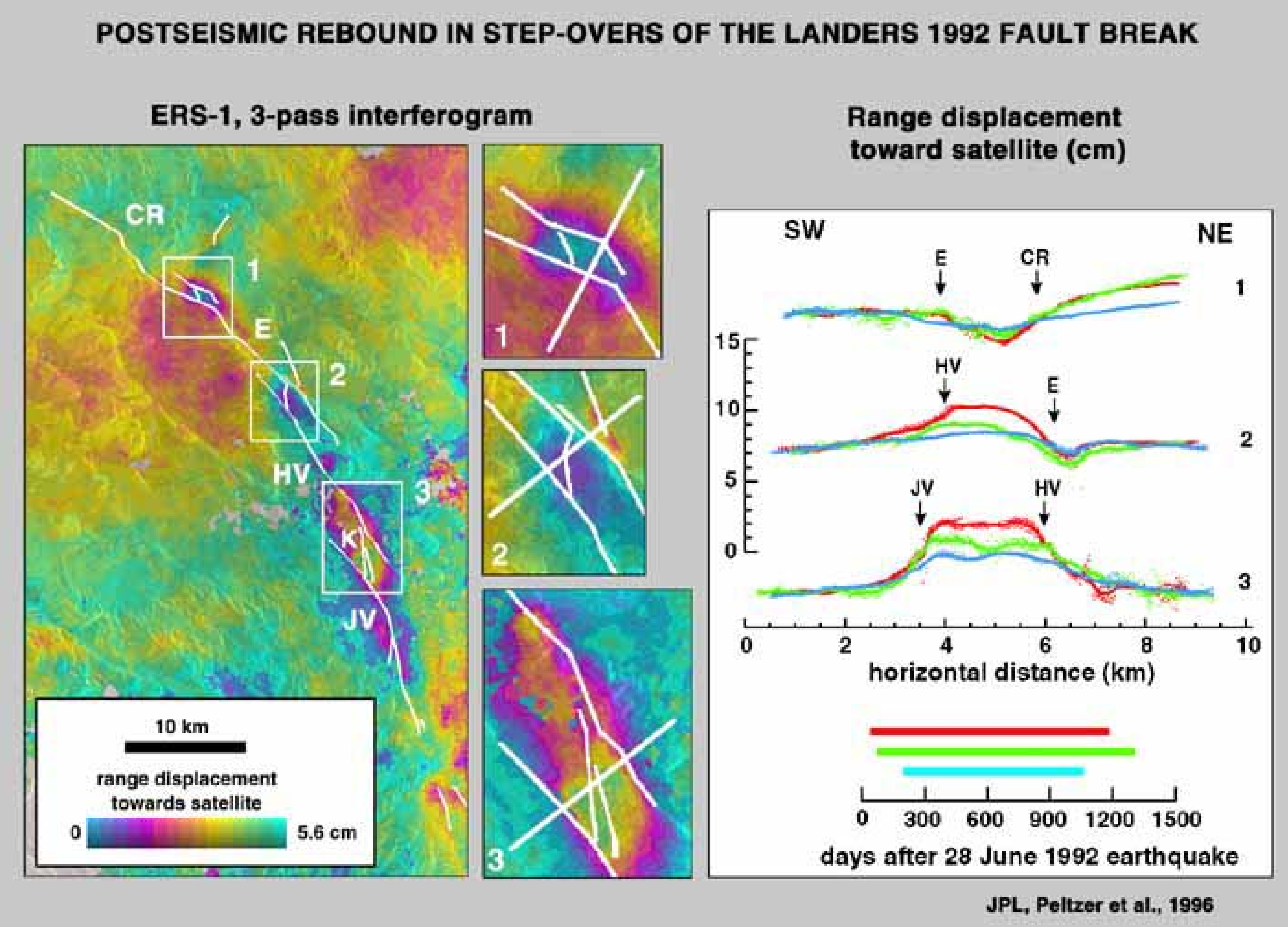


Ge and Stover (2000)

Pore pressure fluctuation due to atmospheric loading



Hosoya and Tokunaga (2003)



Relaxation time ・・・273±44 days (Bosl and Nur, 1998)

Concept of linear isotropic poroelasticity

- Assuming that the rock-masses can be treated as linear isotropic poroelastic materials:
  - Total deformation can be treated as linear combination of deformation due to the external stress plus deformation due to the change of pore pressure (Biot, 1941)
  - The increment of water content in a porous material can be linearly related to the mean external stress and pore pressure (Biot, 1941)

Constitutive relationship and governing equations for linear-isotropic poroelastic material

- Constitutive relationships

$$\varepsilon_{ij} = \frac{1}{2G}\left(\sigma_{ij} - \frac{\nu}{1+\nu}\sigma_{kk}\delta_{ij}\right) + \frac{\alpha}{3K} p\delta_{ij}$$

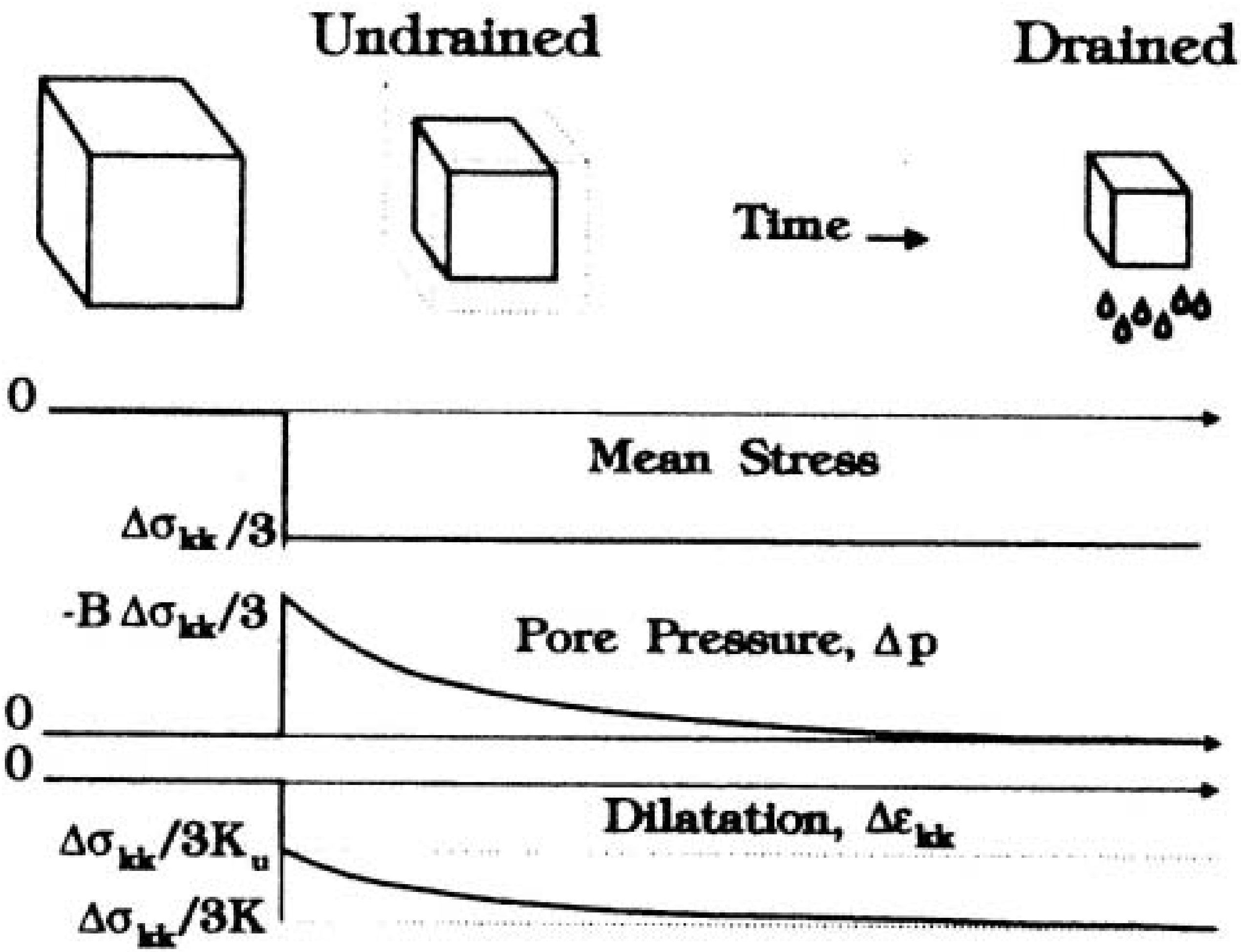
$$\zeta = \frac{\alpha}{K}\frac{\sigma_{kk}}{3} + \frac{\alpha}{KB} p$$

- Governing equations

$$G\nabla^2 u_i + \frac{G}{1-2\nu}\frac{\partial^2 u_k}{\partial x_i \partial x_k} = \alpha\frac{\partial p}{\partial x_i} - F_i$$

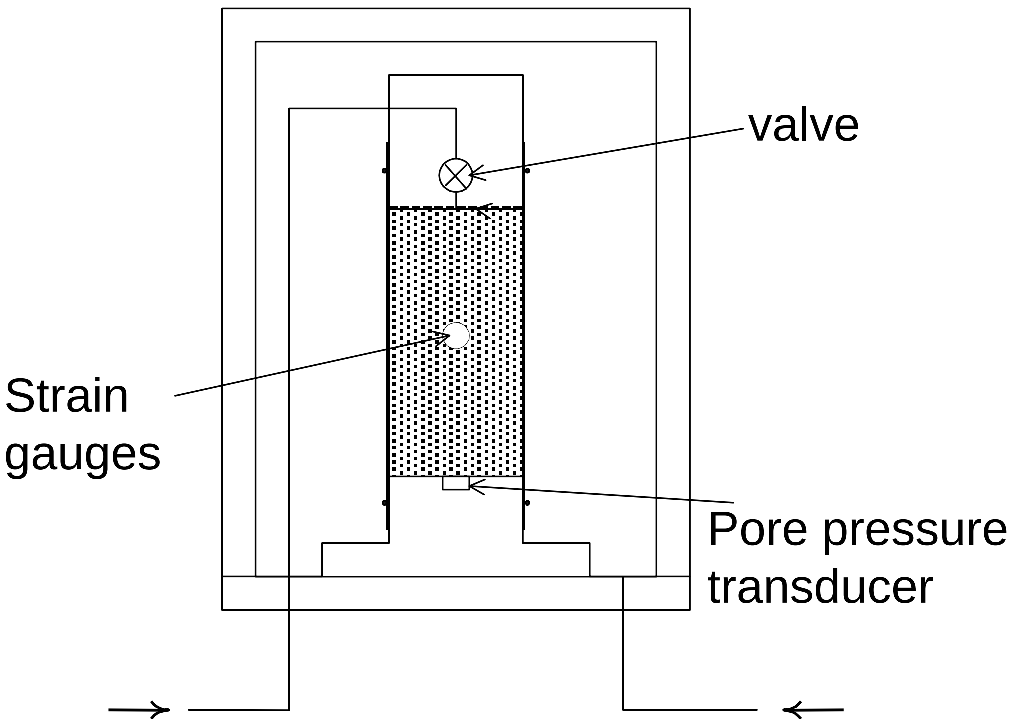
$$\frac{\partial}{\partial x_i}\left(\frac{k_{ij}}{\mu}\frac{\partial p}{\partial x_j}\right) = \frac{\alpha}{KB}\left(\frac{B}{3}\frac{\partial \sigma_{kk}}{\partial t} + \frac{\partial p}{\partial t}\right)$$

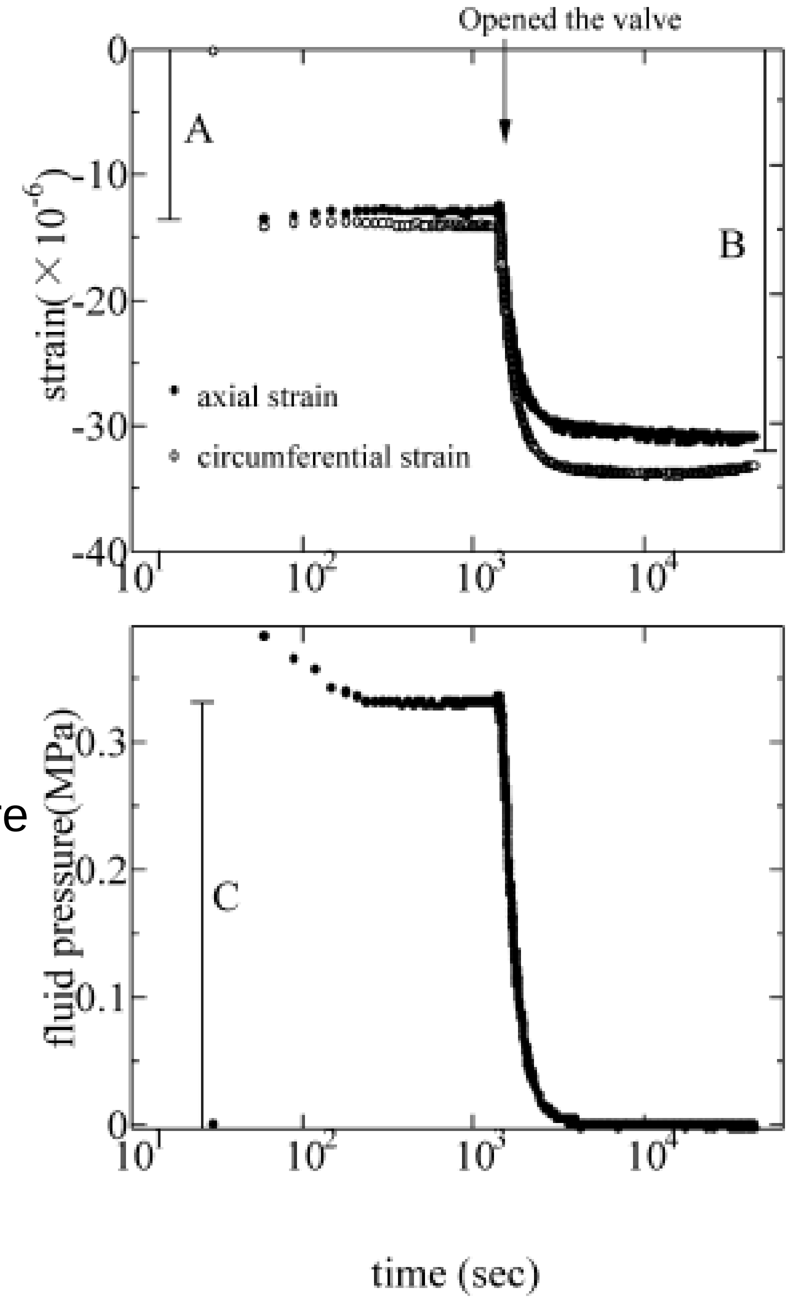
How to understand the poroelastic deformation processes？



Roeloffs (1996)

Laboratory experiments





Tokunaga et al. (2002)

Anisotropy

- Geomaterials are typically anisotropic
- Modeling anisotropic material as an equivalent isotropic one can lead to unexpected and erroneous results
- Theoretical development
  - Biot (1955), Carroll (1979), Thompson and Willis (1991), Cheng (1997)
    - Cheng, A. H. –D., 1997, Material coefficients of anisotropic poroelasticity. Int. J. Rock Mech. Min. Sci., 34 (2), 199-205.
- Laboratory measurements
  - Aoki et al. (1993), Tokunaga et al. (1998), Hart and Wang (1999), Lockner and Stanchits (2002)

Possible anisotropic effect

- “The strains associated with these hydrological influences are not limited to areal strain, and shear strains are also contaminated in the Donalee data at a level of a few hundred nanostrain.” (Gwyther et al., 1996, GRL, 23, 2425-2428)

Suggesting that the material is anisotropic and/or inelastic.

Constitutive relationships

- Stress-strain relations

$$\sigma_{ij} = M_{ijkl}\varepsilon_{kl} - \alpha_{ij}p$$

$$p = M\left(\varsigma - \alpha_{ij}\varepsilon_{ij}\right)$$

- Strain-stress relations

$$\varepsilon_{ij} = C_{ijkl}\sigma_{kl} + \frac{1}{3}CB_{ij}p$$

$$\varsigma = C\left(p + \frac{1}{3}B_{ij}\sigma_{ij}\right)$$

Pore pressure can be generated by incremental shear as well as normal stress, and vice versa.

Example (undrained condition)

$$p = -\frac{1}{3}B_{ij}\sigma_{ij}$$

In the case of transversely isotropic material:

$$p = -\frac{1}{3}\begin{pmatrix} B_1 & B_1 & B_3 \end{pmatrix}\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{pmatrix}$$

$$\sigma_1 + \sigma_3 = 0$$

$$\sigma_2 = 0$$

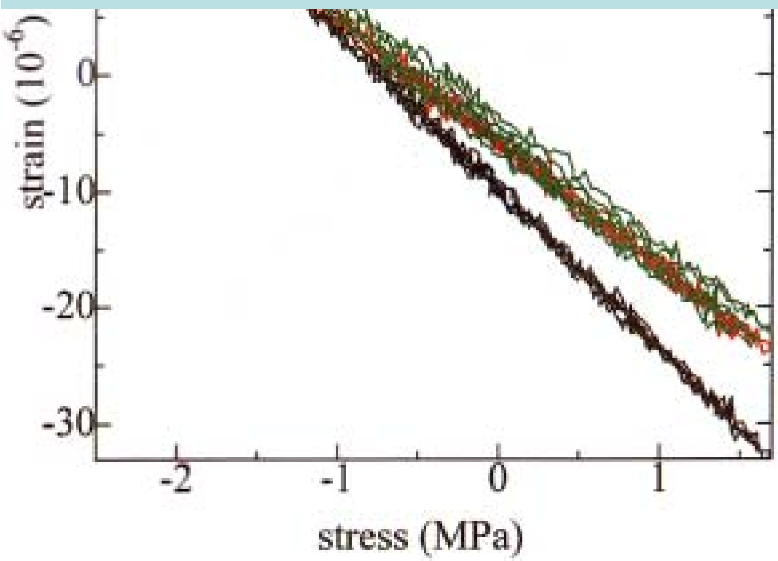
$$p = -\frac{1}{3}\left(B_1 - B_3\right)\sigma_1 \neq 0$$

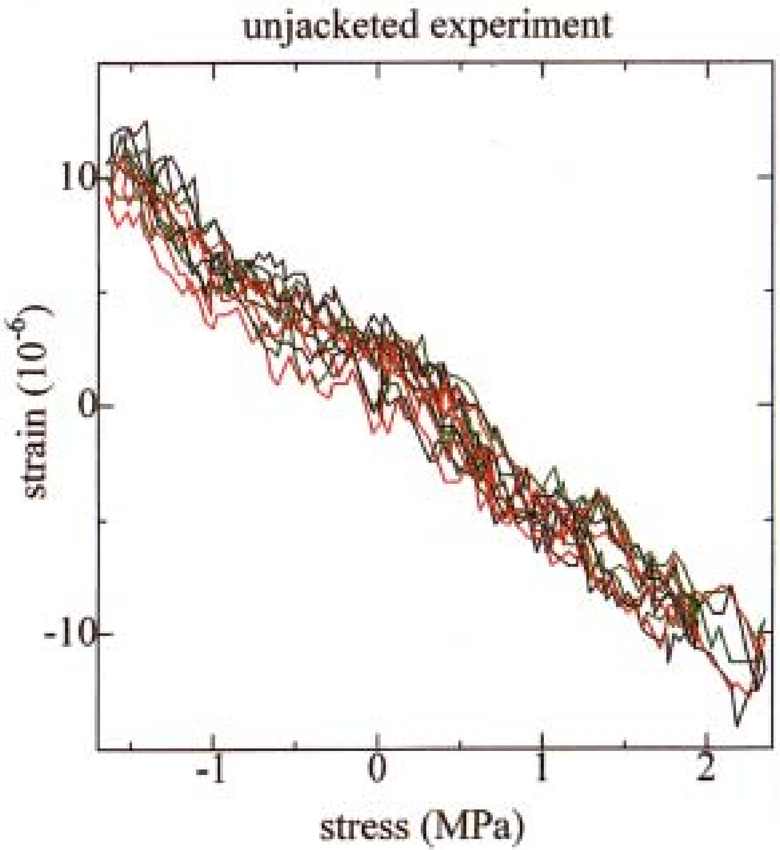
Laboratory measurements of anisotropic poroelastic parameters (Berea sandstone)

The linear unjacketed solid frame compressibility shows very slight anisotropy, thus, it may be reasonable to assume micro-isotropy.

↓

Anisotropy might be due to preferred alignment of pore or microcracks.





(Tokunaga, 2000)

Stress-related change of poroelastic parameters

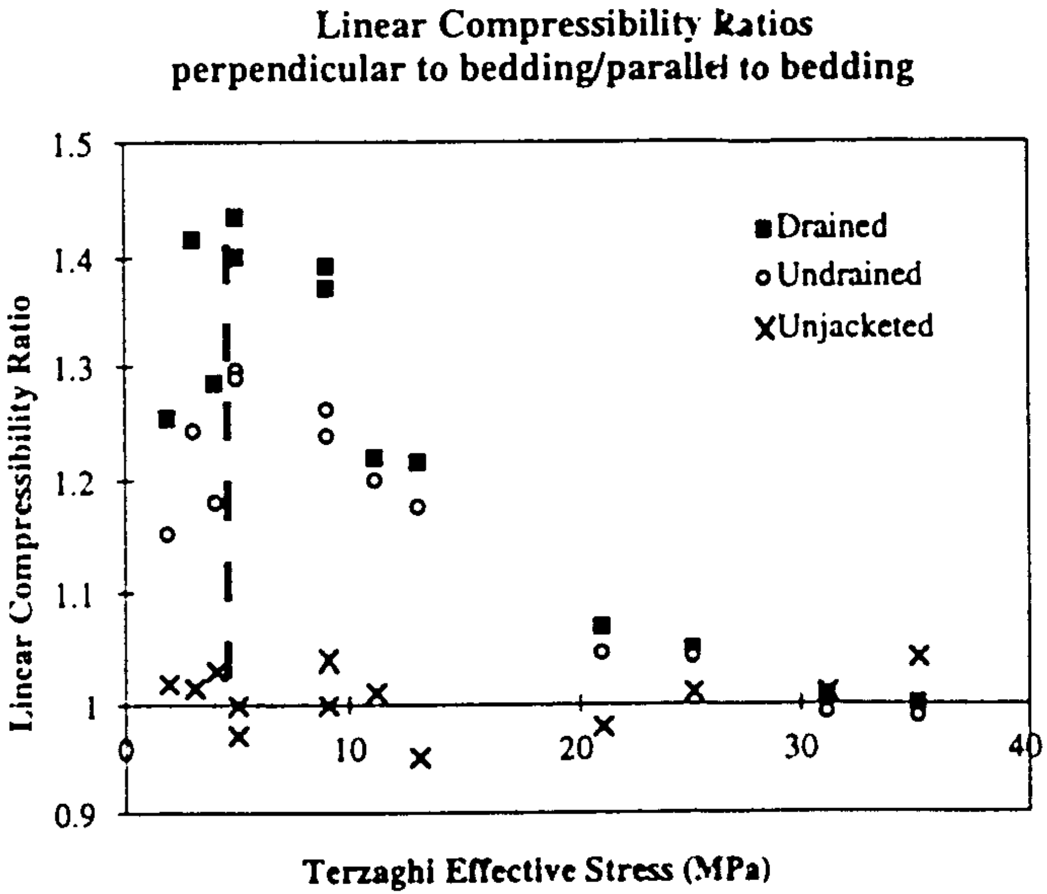


Figure 4. Anisotropy ratios of linear compressibilities perpendicular to bedding over linear compressibilities parallel to bedding. The anisotropy appears to be largest at about 5 MPa, above which it decreases exponentially.

(Hart and Wang, 1999)

Evaluating the degree of anisotropy

$$\varepsilon = \frac{M_{11} - M_{33}}{2M_{33}}$$

$$\delta = \frac{\left(M_{13} + M_{44}\right)^2 - \left(M_{33} - M_{44}\right)^2}{2M_{33}\left(M_{33} - M_{44}\right)}$$

$$\gamma = \frac{M_{66} - M_{44}}{2M_{44}}$$

Cf: isotropic material

$$M_{11} = M_{33}$$

$$M_{66} = M_{44}$$

$$M_{13} = M_{33} - 2M_{44}$$

(Thomsen, 1986)

Degree of anisotropy of several rocks

	ε	δ	γ
Cotton Valley shale	0.135	0.205	0.180
Mesaverde mudstone	0.034	0.211	0.046
Pierre shale	0.015	0.060	0.030
Taylor sandstone	0.110	-0.035	0.255
Dog Creek shale	0.225	0.100	0.345
Marine silty clay	0.000	-0.011	0.160
Berea sandstone (68.95MPa)	0.002	0.020	0.005
Berea sandstone (2〜6MPa)	0.137	0.115	0.042

(Thomsen, 1986; Berge et al., 1991; Tokunaga, 1998)

Conclusions

- By introducing anisotropy, pore pressure can be generated by incremental shear as well as normal stresses, and vice versa.
- Laboratory experimental results suggest that the Berea sandstone is anisotropic at low “effective stress” condition. This anisotropy might be due to preferred alignment of pore or microcracks.
- The anisotropy of the sandstone decreases as the effective stress increases, and the sample behaves isotropically at the higher effective stress.
- It might be necessary to introduce anisotropic poroelastic theory to better understand the deformation-pore pressure coupling problems.