Application of Cross-Spectrum Analysis of the Barometric and Tidal Responses to Determinate the Hydrological Properties of Well-Aquifer System

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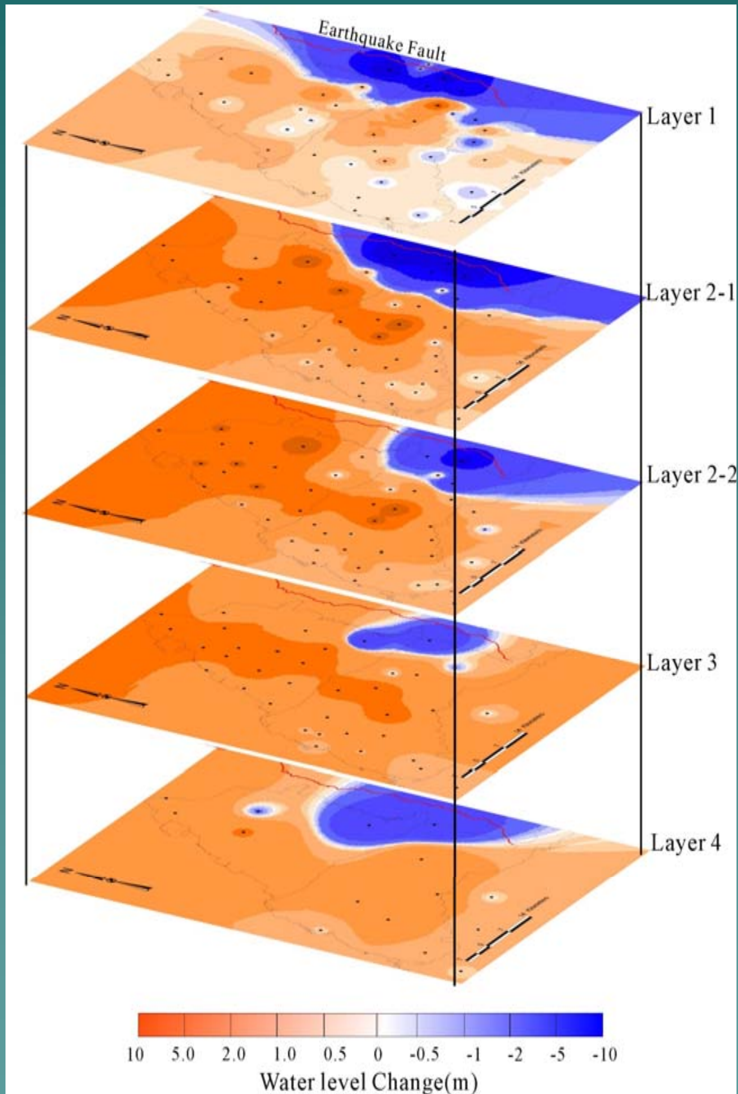
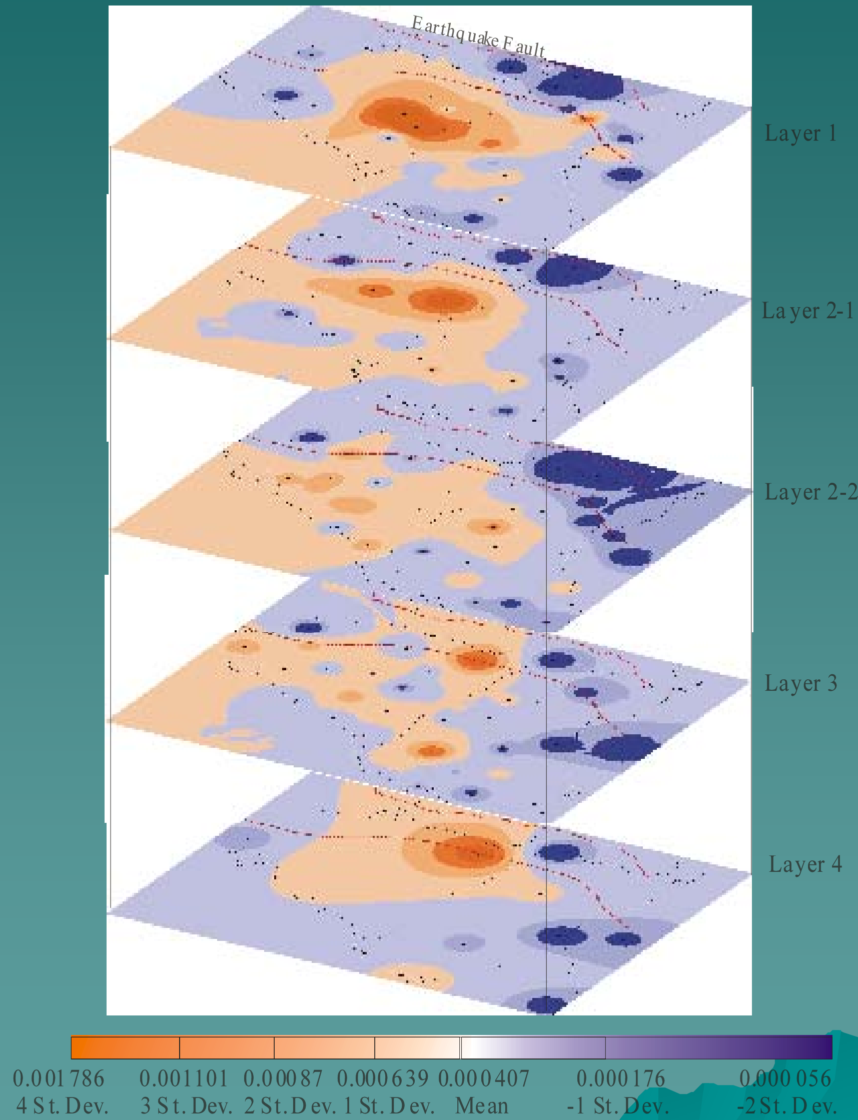
The Effect of the Geological Setting on the Coseismic Groundwater Level Changes

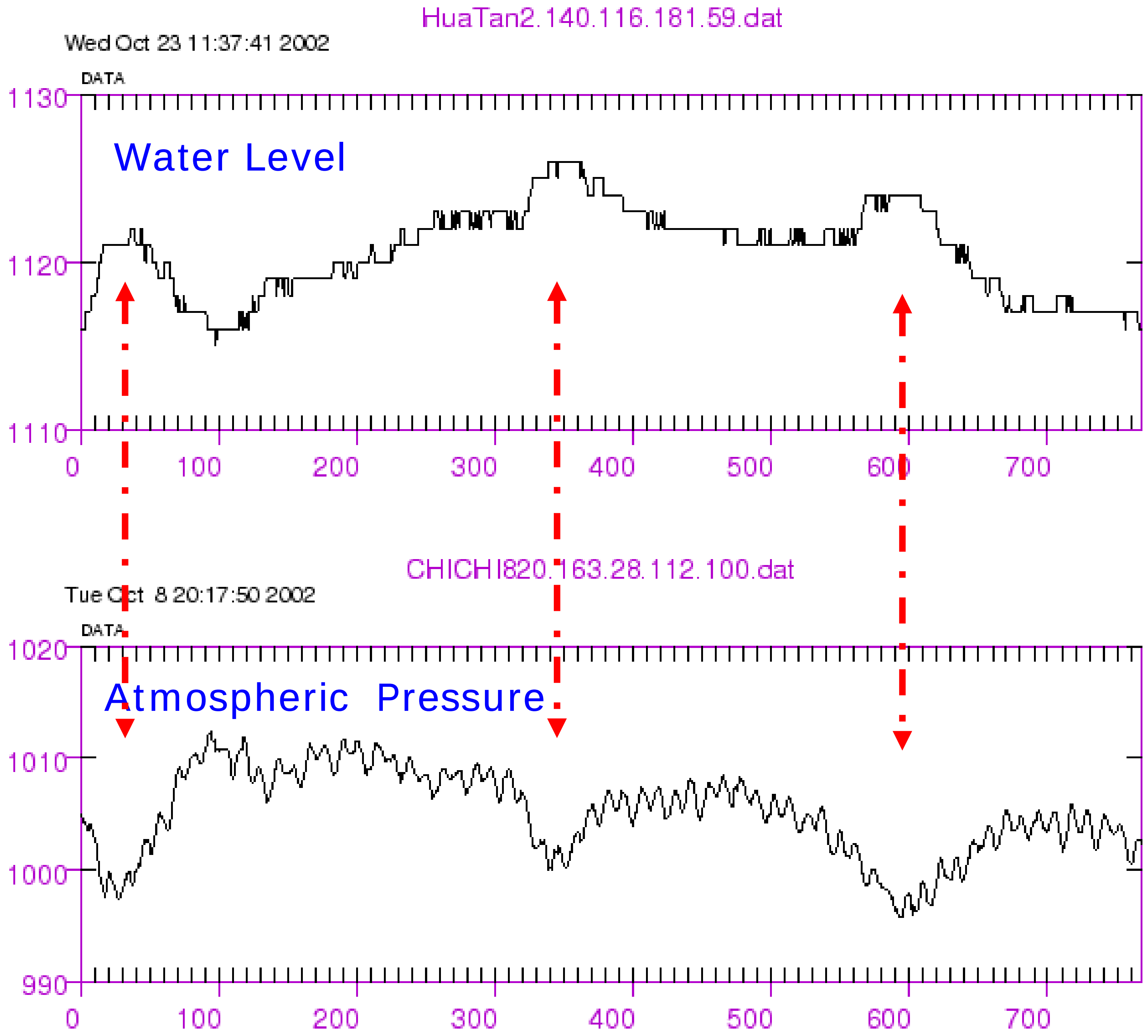


Distribution of coseismic groundwater level changes of the Chi-Chi Earthquake,1999



Distribution of the hydrological conductivity of the Choushuishi Alluvial Fan

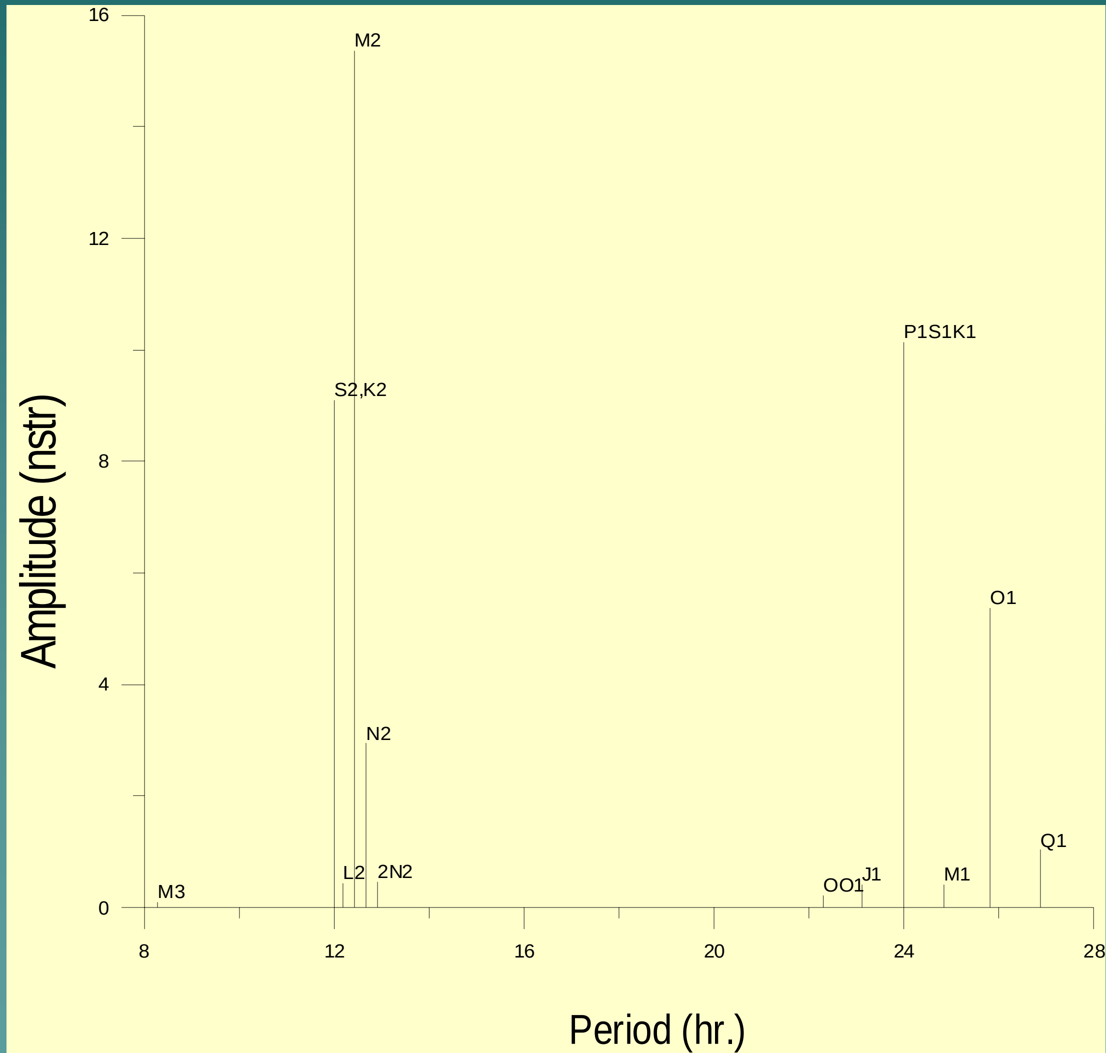
Begin from Original Data



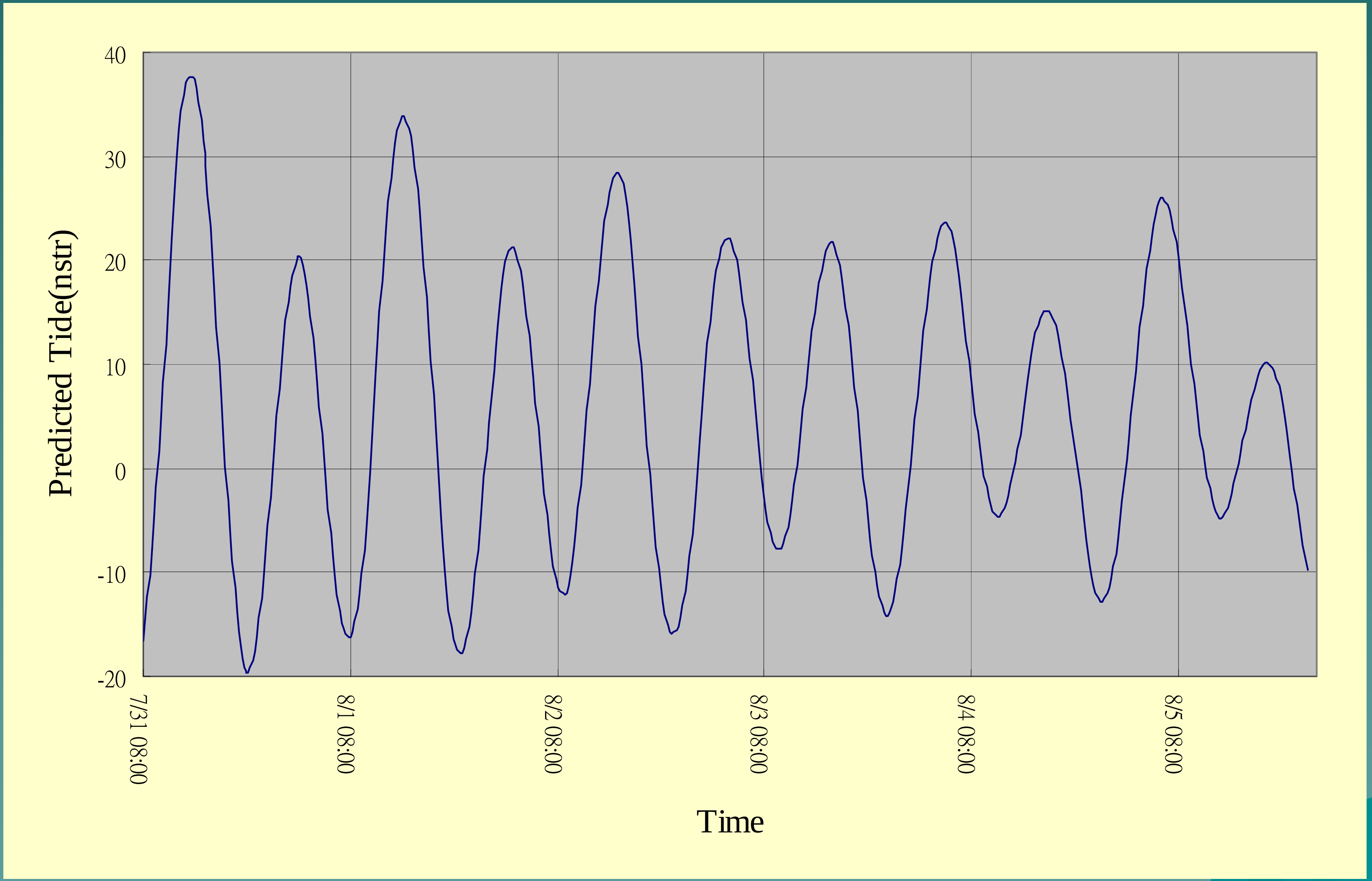
Classified the All Wells

Well no.	name	Barometric	Pumping	Month. Chg.	Stable	Seasonal Chg.	Ocean Tide	Tidal Comp.	Rainfall Effect
7010111	國 聖(一)		o	0.5		o			o
7010121	國 聖(二)		o	1.0		o			o
7010131	國 聖(三)	o	-	0.04	o	-	-		
7010211	東 芳(一)		o	3.0		o			
7010221	東 芳(二)	-		1.0		o			
7020111	洛 津(一)			2.0		o		o	
7020121	洛 津(二)			3.0		o		o	
7020131	洛 津(三)	-		2.0		o	-	o	
7040111	線 西(一)		o	0.8		o			
7040121	線 西(二)		o						
7040131	線 西(三)	-	o	2.0					
7040141	線 西(四)			0.5		o	o		
7050111	全 興(一)	-	-	0.4	o	o	-		
7050121	全 興(二)	-	o	4.0			o		
7050131	全 興(三)			1.0			o		
7050141	全 興(四)			0.6			o		
7060111	文 昌(一)		o	2.0					
7060121	文 昌(二)			4.0		o			
7060131	文 昌(三)		o	4.0		o			
7060141	文 昌(四)			1.6	o	o			
7080111	花 壇(一)		o	2.0				o	
7080121	花 壇(二)	-	-	0.2		-	-		
7080131	花 壇(三)		o	2.0					
7080141	花 壇(四)			0.3	o	o		o	
7100111	員 林(一)		-	1.0	o				
7100121	員 林(二)		o	2.0					
7100131	員 林(三)		o	1.0					
7100141	員 林(四)		o	0.6		o			
7110111	溪湖(一)		o	3.0		o			
7110121	溪 湖(二)		o	2.0		o			

Spectrum of Tidal Potential at Donher (3), 1999/8/1~1999/12/31 (Derived from ETERNA Program)

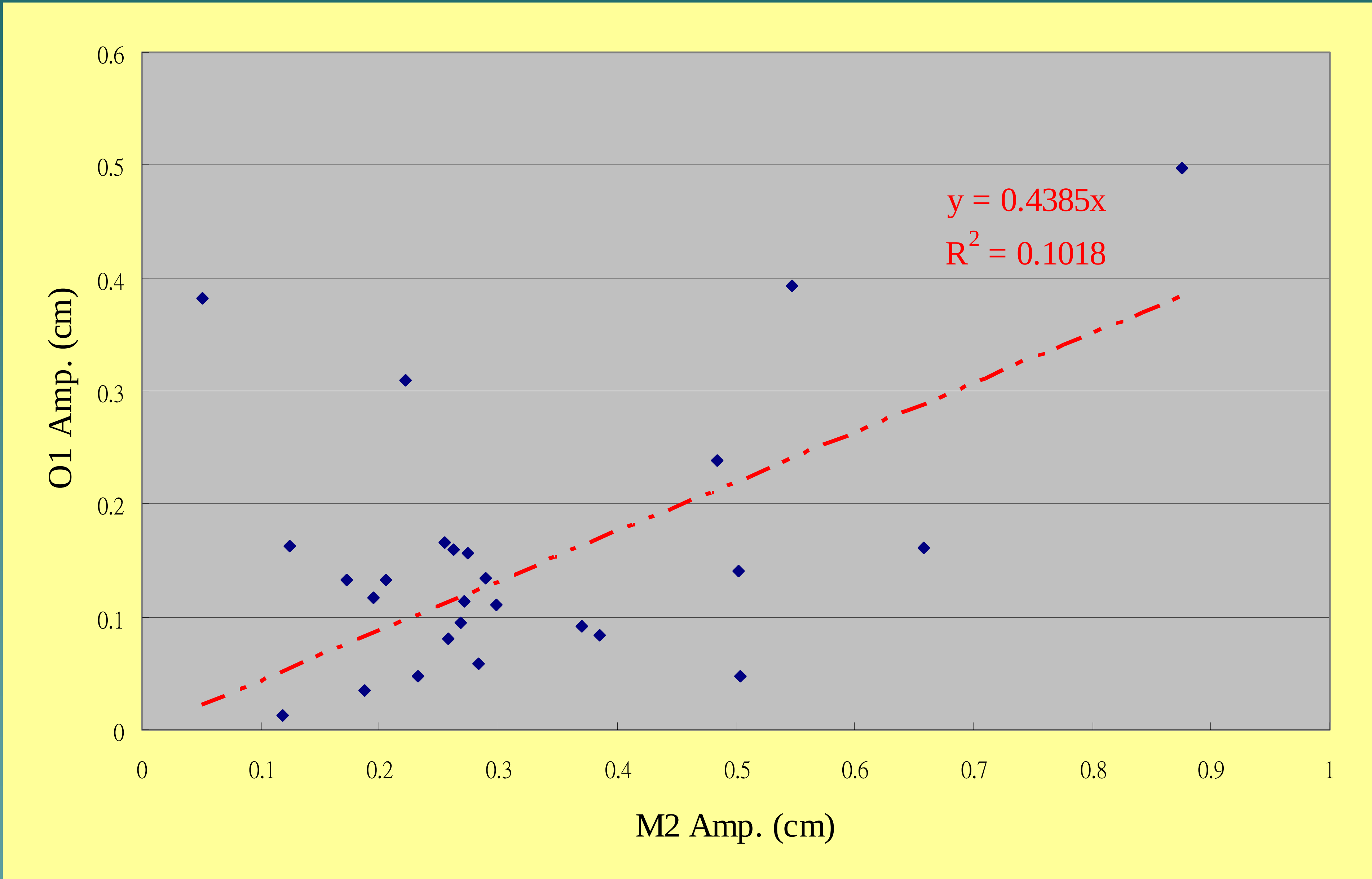


Tidal Potential Series of Donher(3) 1999/8/1~8/5



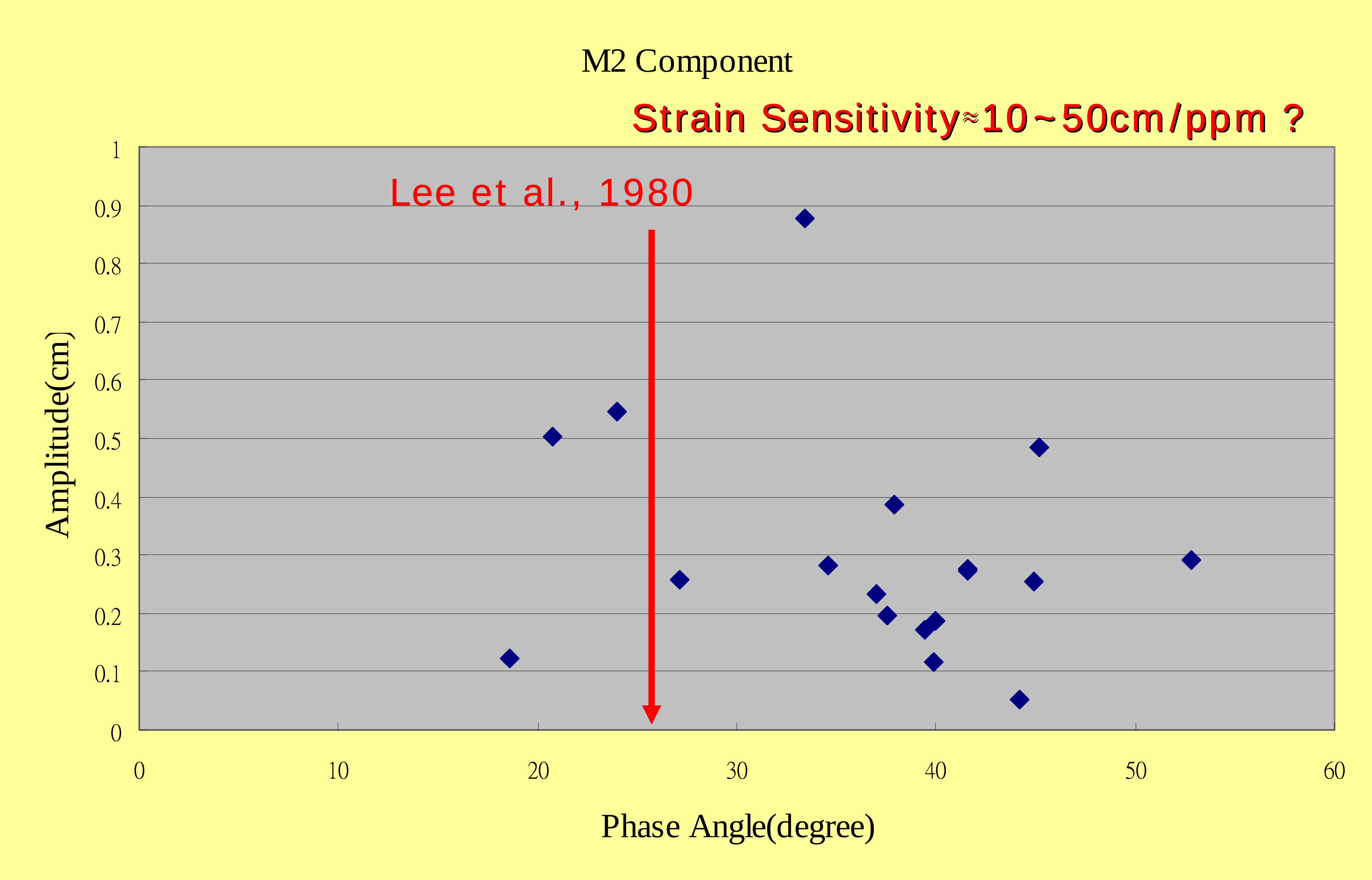
O_1/M_2 Amplitude Ration

(Derived from Baytap-G Program)



M_2 Amplitude/ Phase Angle

(Derived from Baytap-G Program)



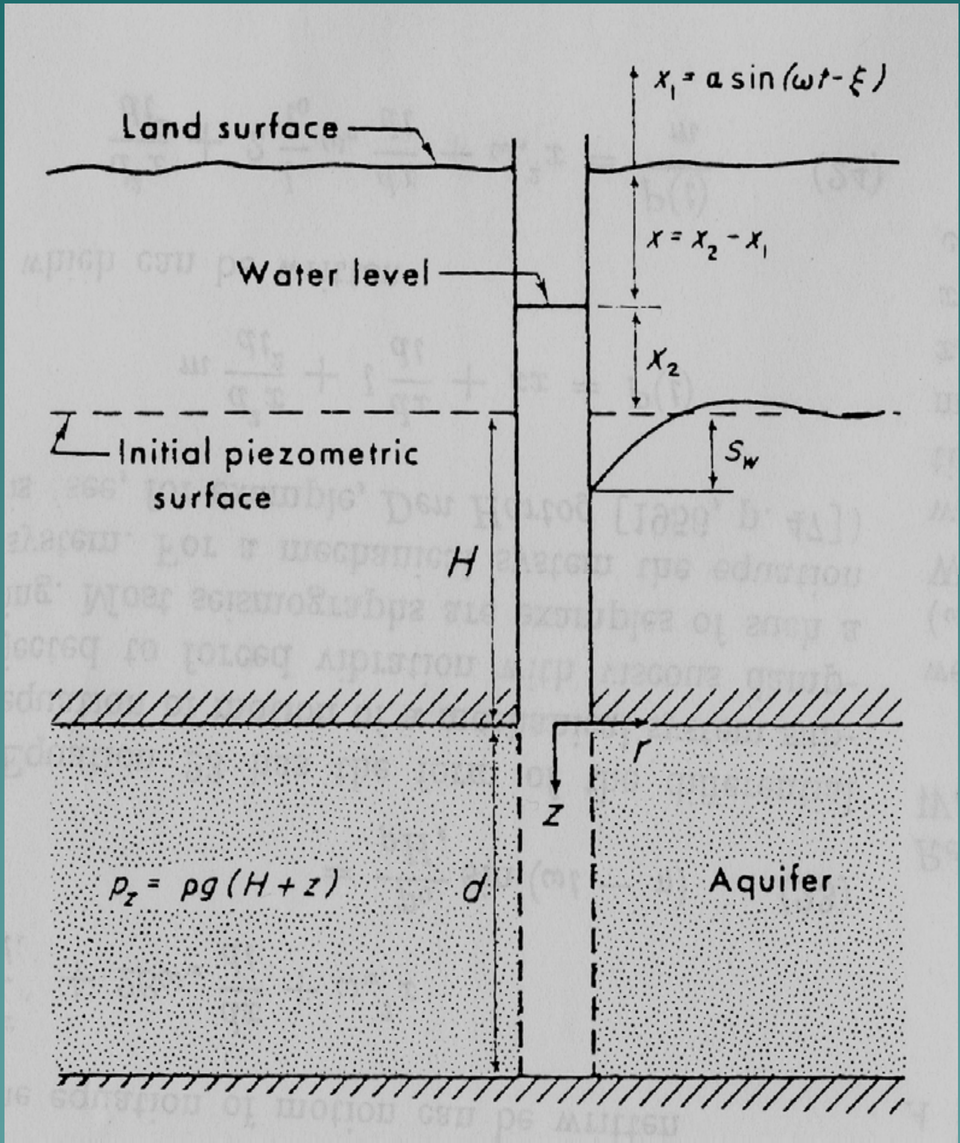
Purposes of the Research

- Clarify the mechanism of the intermediate period fluctuation of groundwater level.

- Determinate the fluid flow properties of well-aquifer system

- Set up the analysis techniques for the groundwater level responses to the different sources.

Well-Aquifer System



(Cooper,1965)

- Confinement of the aquifer (Leakance Problem ?)
- Well or Screen structures Effect (radius, depth, pressure head, thickness)
- Wellbore Storage
- Amplification & Resonance
- Effects of Water Table

Solution: Looking for the Bulk Parameters to representation the behavior of the well-aquifer system

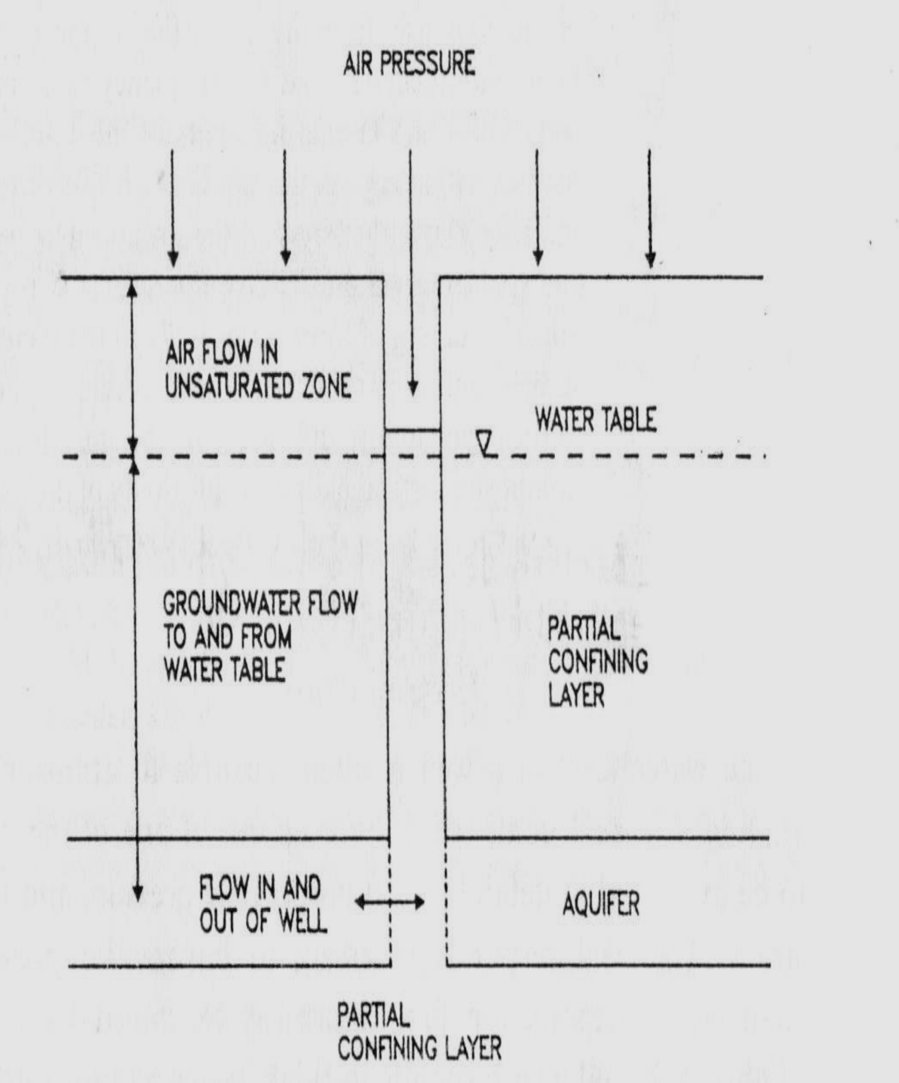
Advantages of the Analysis from Groundwater Level Responses to Atmospheric Loading

- The amplitudes are larger than resolution of the instrument
- The mechanism are more simpler
- With the real observation data (same interval)
- Major Component of the Intermediate period groundwater level Fluctuation

Theoretical response of wells in partially confined aquifers to periodic atmospheric loading

- Previous Work
  - Jacob (1940)
  - Weeks(1979)
  - Rojstaczer(1988a)
  - Igarashi and Wakita(1991)
  - Rojstaczer(1988b)
  - Rojstaczer(1989)
  - Ishimura(2002)

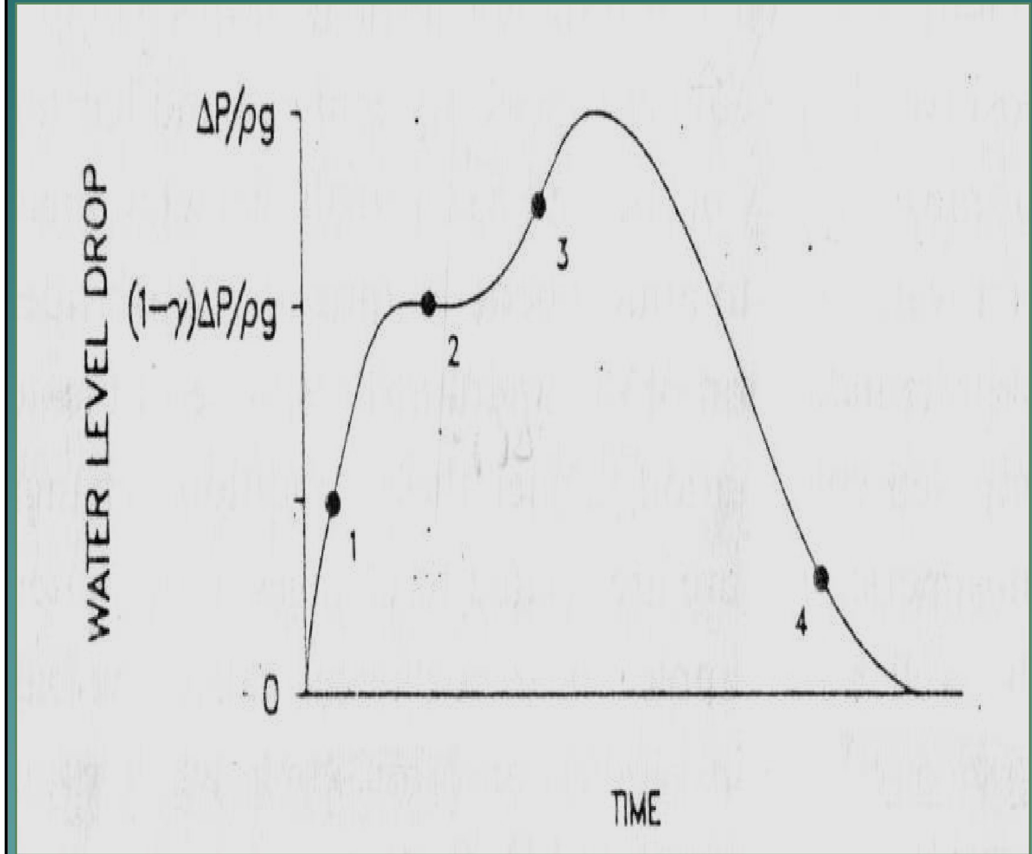
Mechanism of the Groundwater Level Response to the Atmospheric Loading



Rojstaczer(1988a)

- Pressurize on the open borehole
- Surface loading
- Air flow in unsaturated zone
- Groundwater flow to and from water table
- Flow in and out between the well and aquifer

Different Stages of the Groundwater Level Response to the Atmospheric Loading



Rojstaczer(1988a)

Stage 1: Water flow in and out of well driven by the pressure imbalance

Stage 2: Equilibrium with the undrained response

Stage 3: Depressurize due to the potential of pressure imbalance decreasing.

Stage 4: New imbalance between water table and aquifer, groundwater back to aquifer.

Theoretical Assumption (Rojstaczer, 1988a)

- The undrained response of the aquifer and confining layer to surface loading are the same
- Air flow between the ground surface and the water table and the groundwater flow in the confining layer are vertical.
- The groundwater flow between the aquifer and the borehole is horizontal

So the theoretical response to the Atmospheric Loading can be simplify into three 2D flow problems.

- Vertical air flow in unsaturated zone
- Vertical groundwater flow in partial confining layer
- Horizontal groundwater flow between the aquifer and the borehole

Vertical Air Flow Between Earth's Surface and the Water Table

Buckingham(1904), Weeks(1979):

$$D_a \partial^2 P_a / \partial Z^2 = \partial P_a / \partial t$$

Boundary Condition: $P(-L,t) = A\cos(\omega t)$

$$P(L,t) = A\cos(\omega t)$$

The Solution is given by(Rojstaczer,1988b):

$$P_a = (M - iN) A \exp(i\omega t)$$

$$M = \frac{2\cosh(\sqrt{R})\cos(\sqrt{R})}{\cosh(2\sqrt{R}) + \cos(2\sqrt{R})} \qquad N = \frac{2\sinh(\sqrt{R})\sin(\sqrt{R})}{\sinh(2\sqrt{R}) + \sin(2\sqrt{R})}$$

$$R = L^2 \omega / 2D_a$$

R: Dimensionless frequency referenced to the unsaturated zone

Vertical Groundwater Flow Between the Water Table and the Aquifer

Rojstaczer (1988b)

$$D\partial^2 P / \partial Z^2 = \partial P / \partial t$$

Boundary Condition: $P(0,t) = MA\cos(\omega t) + NA\sin(\omega t)$

$P(\infty,t) = A\gamma\cos(\omega t)$

The Solution is given by(Rojstaczer,1988b):

$$P = (M + iN - \gamma)A\exp(-(i+1)(0.5qS') \bullet \exp(i\omega t) + A\gamma\exp(i\omega t)$$

$$q = b'\omega / K'$$

$$Q = qS'/2 = b'^2\omega / 2D$$

Q: <u>Dimensionless frequency referenced to the confining layer</u>

Flow Between the Borehole and the Aquifer

Jacob (1946)

$$D\partial^2 p / \partial z^2 = \partial p / \partial t + \omega\gamma A \sin \omega t$$

Boundary Condition: $s(\infty, t) = 0$

$$\lim_{r \to 0} \frac{r \partial s}{\partial r} = \frac{\omega r_w^2 x_0}{2kb} \sin \omega t$$

The Solution is given by(Rojstaczer,1988 b):

$$S_w = i0.5 W x_0 K_0 \left\{ \left[W^2 (S^2 + 1/q^2) \right]^{0.25} \right\} \bullet \exp \left[i0.5 \left\{ \tan^{-1}(qS) \right\} \right] \} \exp(i\omega t)$$

K_0: Modified Bessel Function of the second kind of order zero

$$W = \omega r_w^2 / Kb$$

W: <u>Dimensionless frequency referenced to the aquifer</u>

Response of A Well to Atmospheric Loading: General Case

Air Pressure Change

Drawdown at the well

$$x_0 = \frac{-A}{\rho g} + \frac{P_0}{\rho g} - s_0$$

$$p_0 = p\exp(-i\omega t)$$

$$s_0 = s_w\exp(-i\omega t)$$

Initial Condition

$$BE(\omega) = \left|\frac{x_0 \rho g}{A}\right| = \left|\frac{p_0 - A - s_0 \rho g}{A}\right|$$

$$\theta(\omega) = \arg\left(\frac{x0\rho g}{A}\right)$$

Theorbaro.m

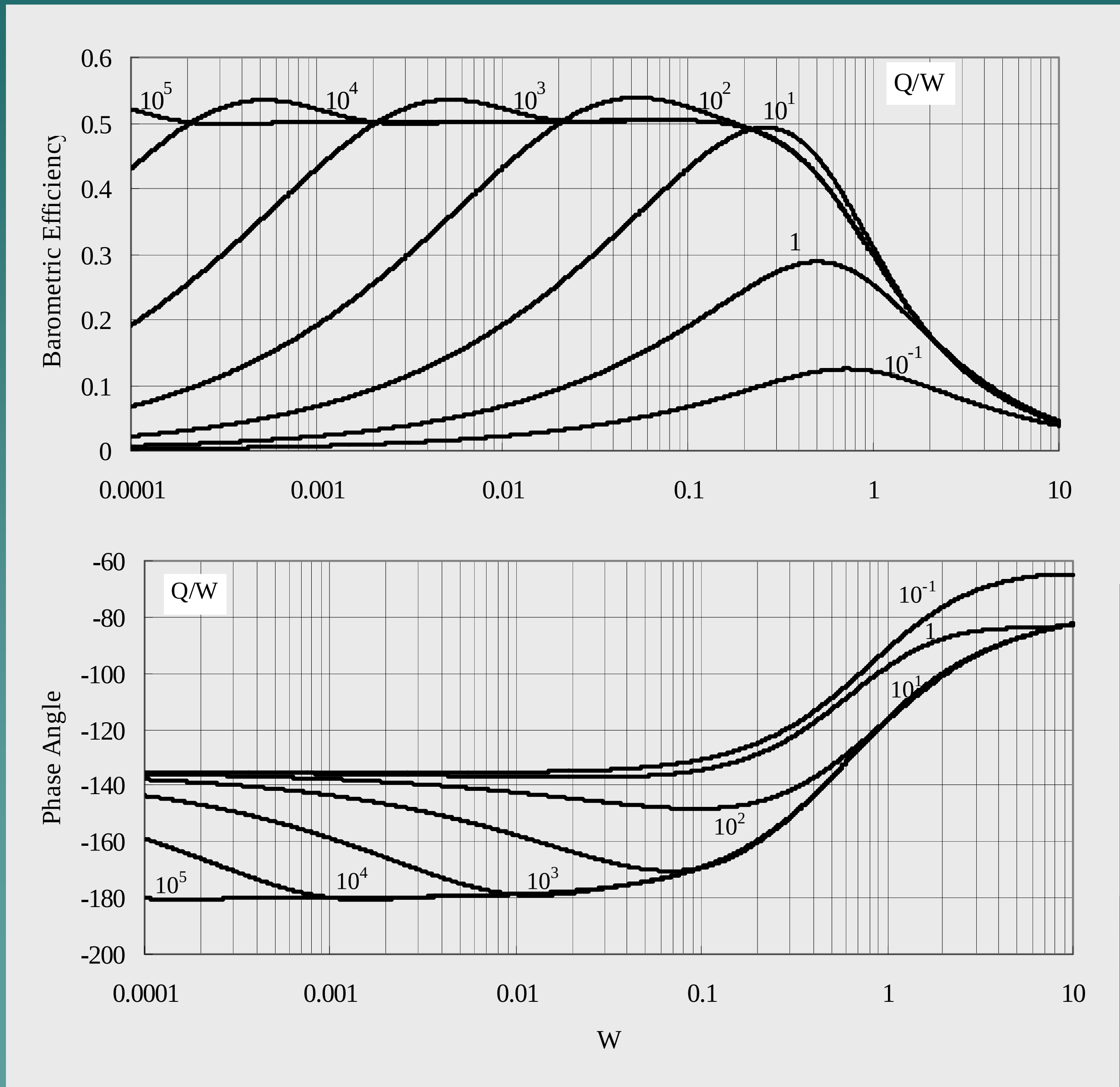
```
%Theorical Barometric Response
clear all
%Parameter Setting
A=1;                    %Air Pressure Change
%Unsaturated Zone
L=10;                   %Depth of the Water Table
Da=0.0001;              %Pneumatic Diffusivity
%Confining Layer
bc=222;                 %Thickness
kc=0.00000001;          %Hydraulic Conductivity
Sc=0.00001;             %Storage Coefficient
kama=0.5;               %Loadind Efficiency
D=0.001;                %Vertical Hydraulic Diffusivity
%Aquifer
b=30;                   %Thickness
K=0.00001;              %Hydraulic Conductivity
rw=0.0762;              %Radius of the Well
S=0.00001;              %Storage Coefficient
i=1;
f=0.000000001;
while f<0.0168;
   R=f*L^2/2*Da;
   Pa(i)=R;
M=2*cosh(R^0.5)*cos(R^0.5)/cosh(2*R^0.5)+cos(2*R^0.5);
N=2*sinh(R^0.5)*sin(R^0.5)/cosh(2*R^0.5)+cos(2*R^0.5);
   q=f*bc/kc;
   p=(M+j*N-kama)*A*exp(-(j+1)*(0.5*q*Sc)^0.5)+A*kama;
   P(i)=p;

 w=f*rw^2/(K*b);
   W(i)=w;
z=j*0.5*w*bessely(0,(w^2*(S^2+1/q^2))^0.25*exp(j*0.5*(atan(q*S))));
   Sw(i)=z;
   Xo(i)=(P(i)-A)/(1+Sw(i));
   be=Xo(i)/A;
   BE(i)=abs(be);
   phase(i)=angle(be);
end
F=F';
CPD=CPD';
BE=BE';
phase=phase';
Barometric=[F CPD BE phase];
csvwrite('C:\work\barometric\Barometric.csv',Barometric);
%plot(CPD,BE);
%plot(CPD,phase);
```

Output

```
%Theorical Barometric Response
clear all
%Parameter Setting
Sc=0.00001;
kama=0.5;
A=1;
b=30;
K=0.00001;
rw=0.0762;
S=0.00001;
ration=1.0;              %Q/W Ration
%
i=1;
f=0.0000001;
while f<16.8;
   w=f*rw^2/(K*b);
   W(i)=w;
   q=2*ration*w/Sc;
   R=0.0001*ration*w;
   %R=f*L^2/2*Da;
   M=2*cosh(R^0.5)*cos(R^0.5)/(cosh(2*R^0.5)+cos(2*R^0.5));
   N=2*sinh(R^0.5)*sin(R^0.5)/(cosh(2*R^0.5)+cos(2*R^0.5));
   %q=f*bc/kc;
   p=(M+j*N-kama)*A*exp(-(j+1)*(0.5*q*Sc)^0.5)+A*kama;
   P(i)=p;
   z=j*0.5*w*bessely(0,(w^2*(S^2+1/q^2))^0.25*exp(j*0.5*(atan(q*S))));
   Sw(i)=z;
   Xo(i)=(P(i)-A)/(1+Sw(i));
   be=Xo(i)/A;
   BE(i)=abs(be);
   phase(i)=angle(be);
   F(i)=f;
   CPD(i)=f*3600*24;
   f=f*1.01;
   i=i+1;
end
F=F';
CPD=CPD';
W=W';
BE=BE';
phase=phase';
Barometric=[F CPD W BE phase];
csvwrite('C:\work\barometric\Barometric.csv',Barometric);
plot(W,BE);
%plot(W,phase);
```

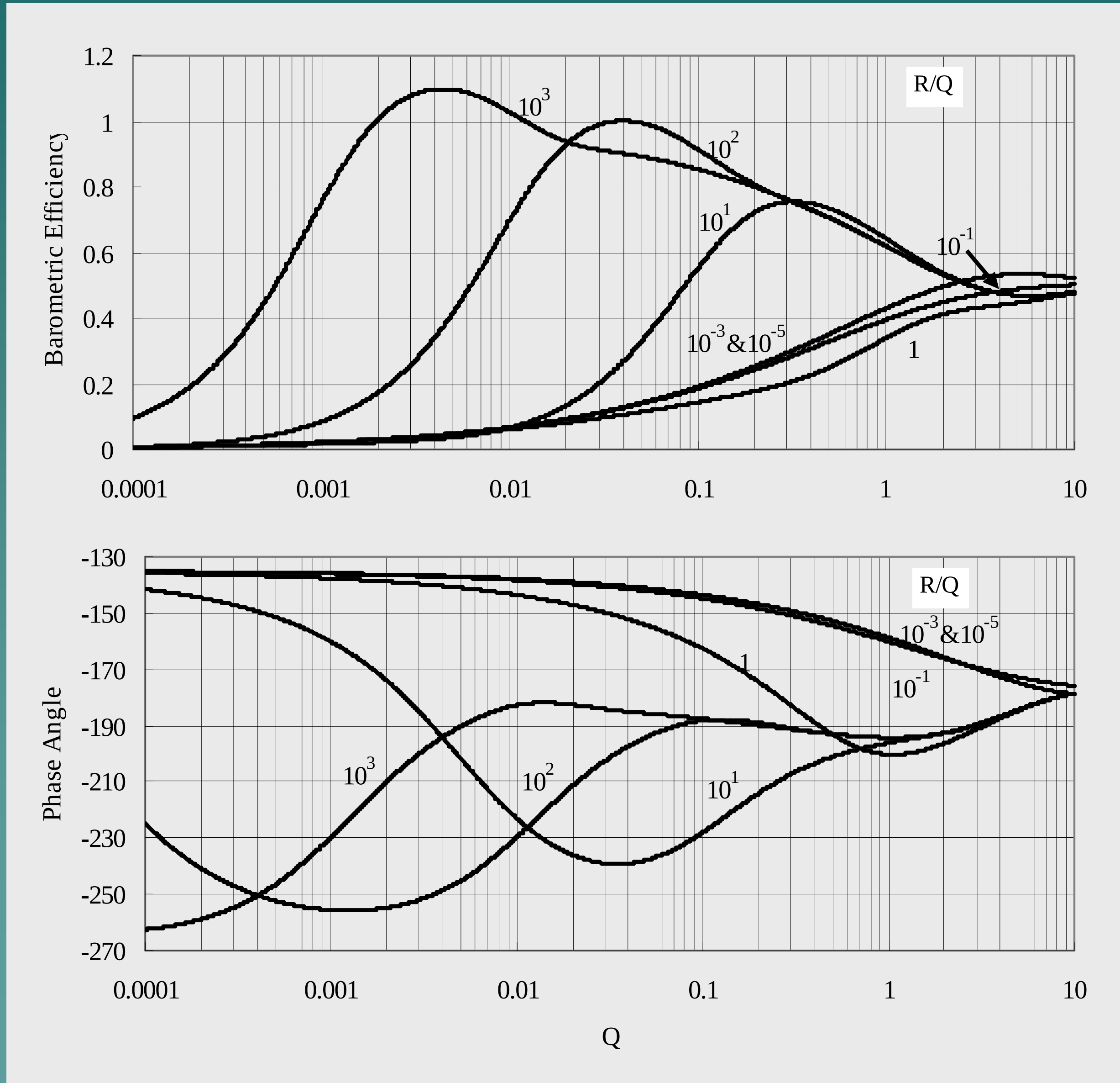
Case I: S,S'equal 0.0001, γ =0.5 in different Q,W ration

- Barometric Efficiency
- Phase Angle in Degree



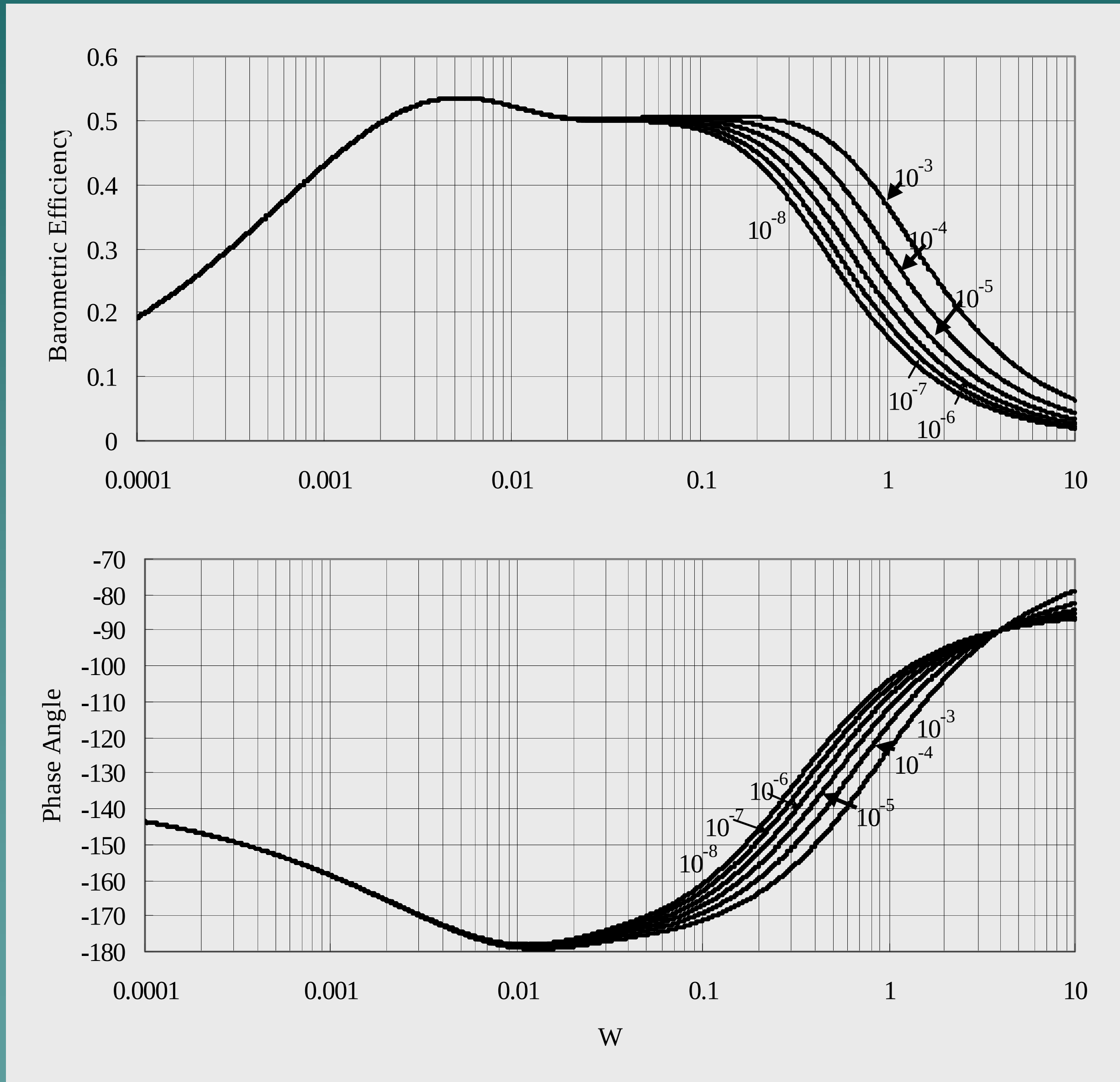
Case II: S,S'equal 0.0001, γ =0.5 in different R,Q ration

- Barometric Efficiency
- Phase Angle in Degree



Case III: γ =0.5 in different S', S ration

- Barometric Efficiency
- Phase Angle in Degree



Application of theorical response

1. Barometric Response of the Water Level

↓

2. Theoretical Response

↓ Curve Fitting (type curve) → R,Q,W

3. Determinate of Fluid Flow Properties
   - Vertical Pneumatic Diffusivity of the Unsaturated Zone
   - Vertical Hydraulic Diffusivity of the Confining Layer
   - Lateral Permeability of the Aquifer

How to Get the Transfer Function? (Rojstaczer,1988b)

- Complex Linear Equation of the water level response

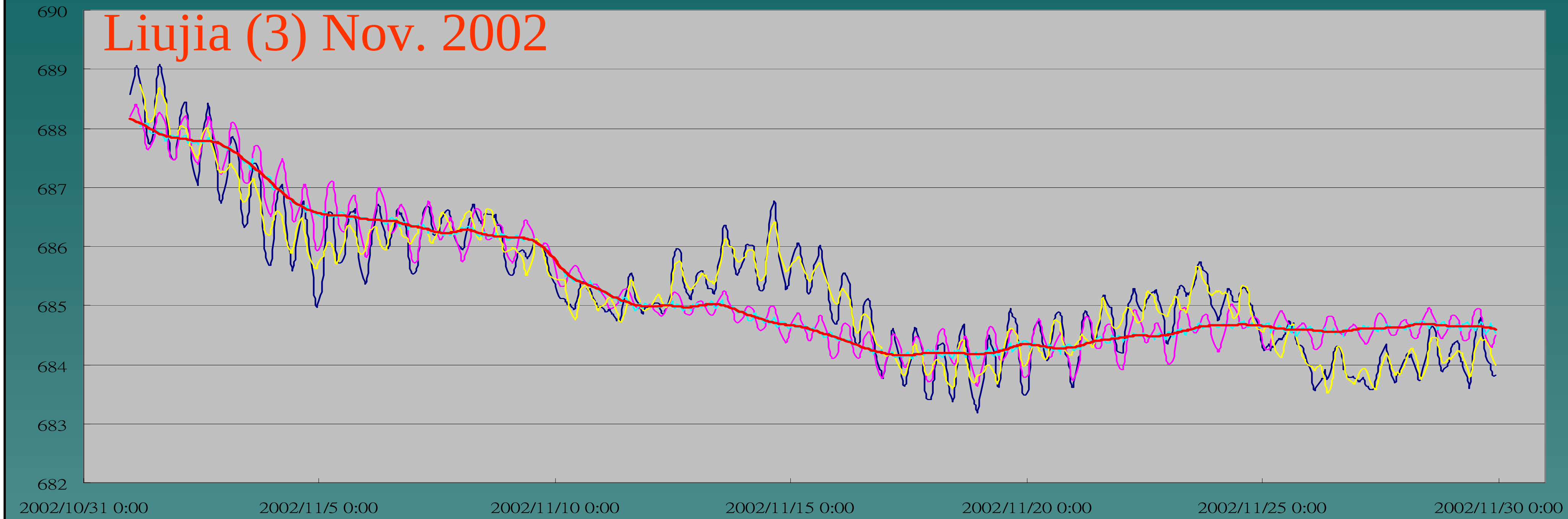
$$\begin{bmatrix} BB & BT \\ TB & TT \end{bmatrix} \begin{bmatrix} HB \\ HT \end{bmatrix} = \begin{bmatrix} BW \\ TW \end{bmatrix}$$

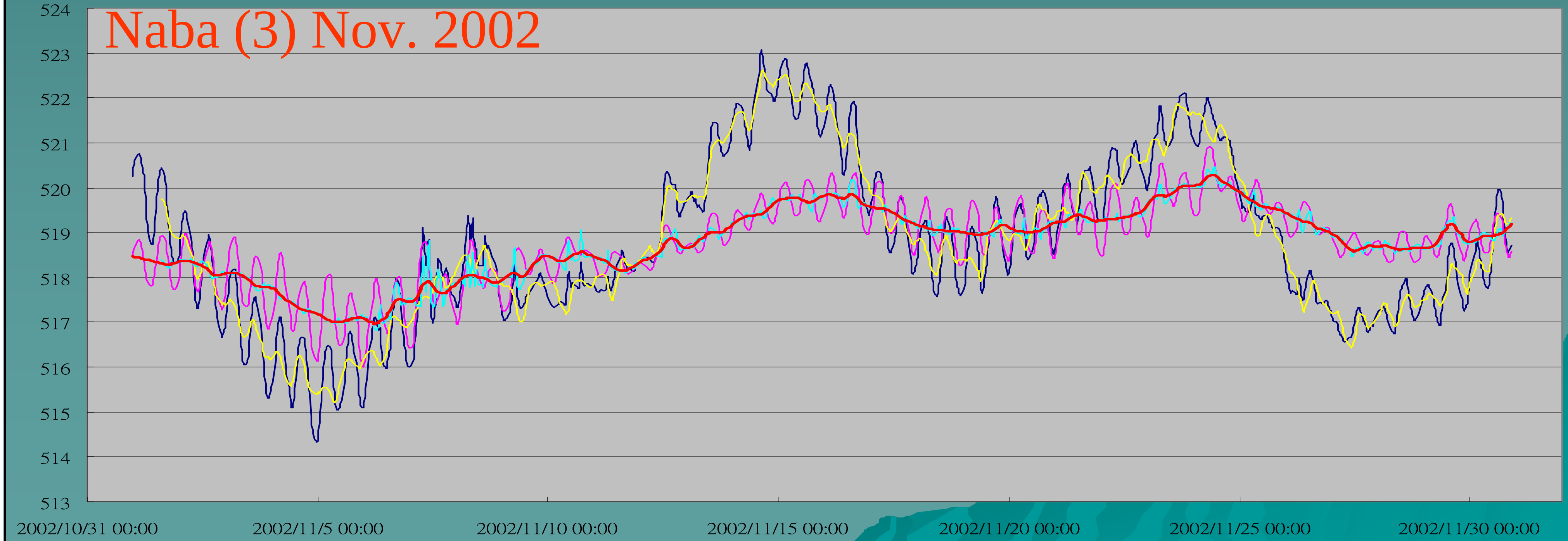
- Removing the long term trend with Zero-Phase High-Pass Filter,Using $5*10^{-9}$Hz as criteria, to calculate the 40 days long-tern trend
- Using Blackman-Tukey Procedures to estimate the Power Spectrum of atmospheric pressures(BB), earth tide(TT) and Cross Spectrum between each other(BT, TB, BW, TW)
- Solving the complex linear equations for every frequency

Taketranf.m

```
addpath ('C:\work\download\matlab');
brec=detrend(bre);
wlc=detrend(wl);
tidec=detrend(tide);
[BB,TT,BT,BTcoh,BTpha,BTfreq]=func_coherence(brec,tidec,696,0.0002777778,348,5);
TB=conj(BT);
crossspectBT=[BB,TT,BT,TB,BTcoh,BTpha,BTfreq];
csvwrite('C:\work\Xspect_BT.csv',crossspectBT);
[BB,WW,BW,BWcoh,BWpha,BWfreq]=func_coherence(brec,wlc,696,0.0002777778,348,5);
crossspectBW=[BB,WW,BW,BWcoh,BWpha,BWfreq];
csvwrite('C:\work\Xspect_BW.csv',crossspectBW);
[TT,WW,TW,TWcoh,TWpha,TWfreq]=func_coherence(tidec,wlc,696,0.0002777778,348,5);
crossspectTW=[TT,WW,TW,TWcoh,TWpha,TWfreq];
csvwrite('C:\work\Xspect_TW.csv',crossspectTW);
for i=1:348;
    Trans=inv([BB(i) BT(i);TB(i) TT(i)])*[BW(i);TW(i)];
    HB(i)=Trans(1);
    HT(i)=Trans(2);
end
HB=HB';
HT=HT';
transpect=[HB HT];
csvwrite('C:\work\transfuc.csv',transpect);
   HBgain=abs(HB);
   HBphase=angle(HB);
   HBspect=[HBgain HBphase];
csvwrite('C:\work\HBspect.csv',HBspect);
   HTgain=abs(HT);
   HTphase=angle(HT);
   HTspect=[HTgain HTphase];
csvwrite('C:\work\HTspect.csv',HTspect);
```

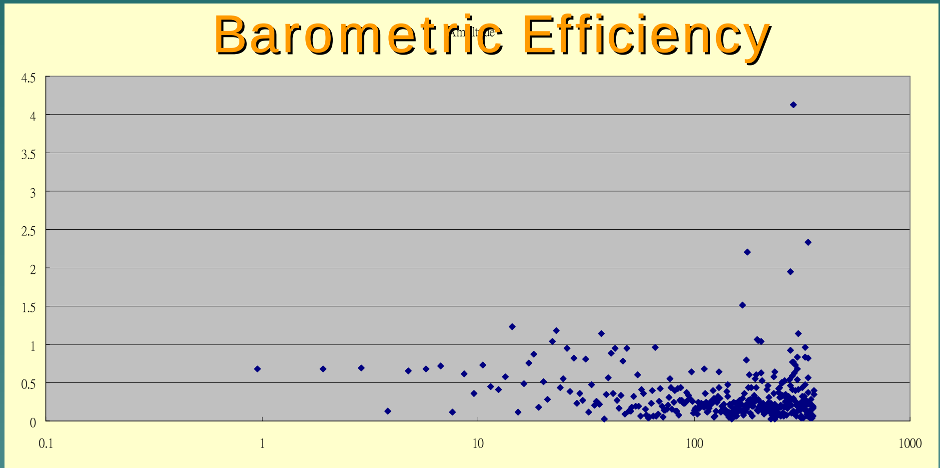
Observation Data

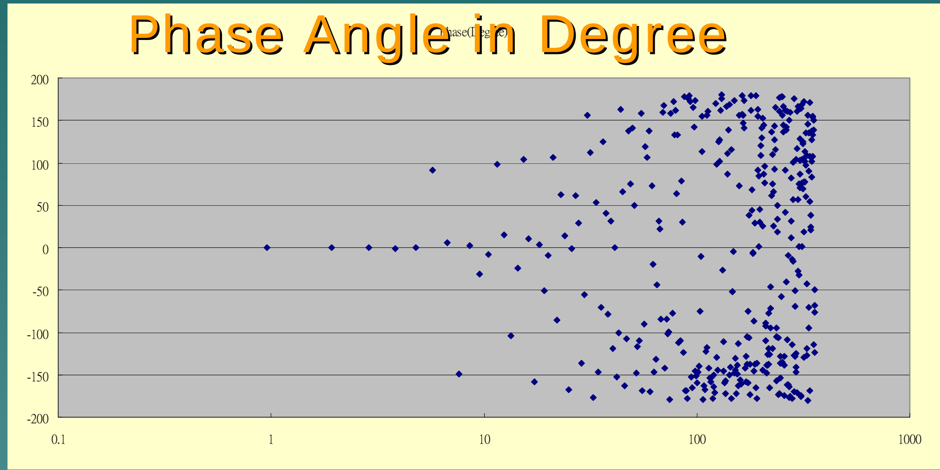




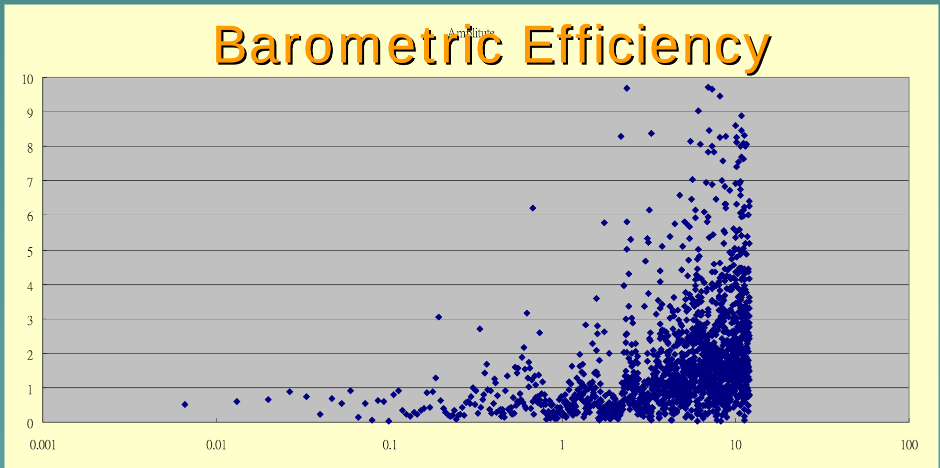
Results Without Moving Long-Term Trend

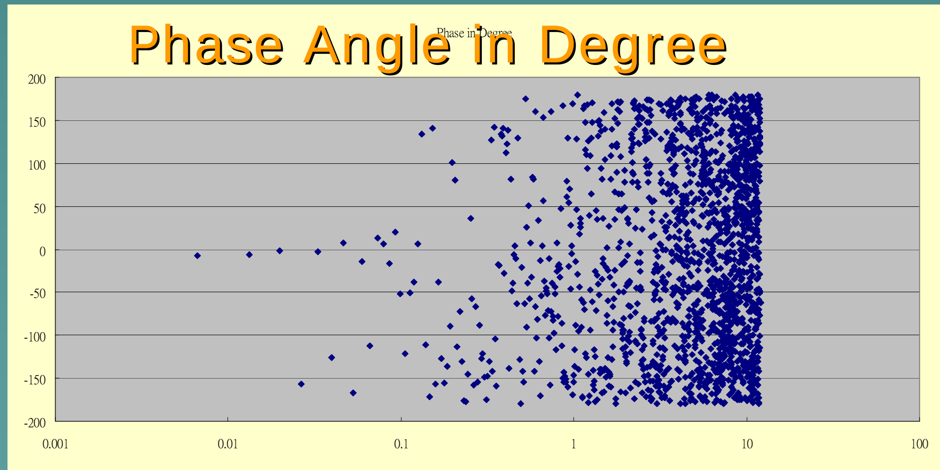
Hourly Data



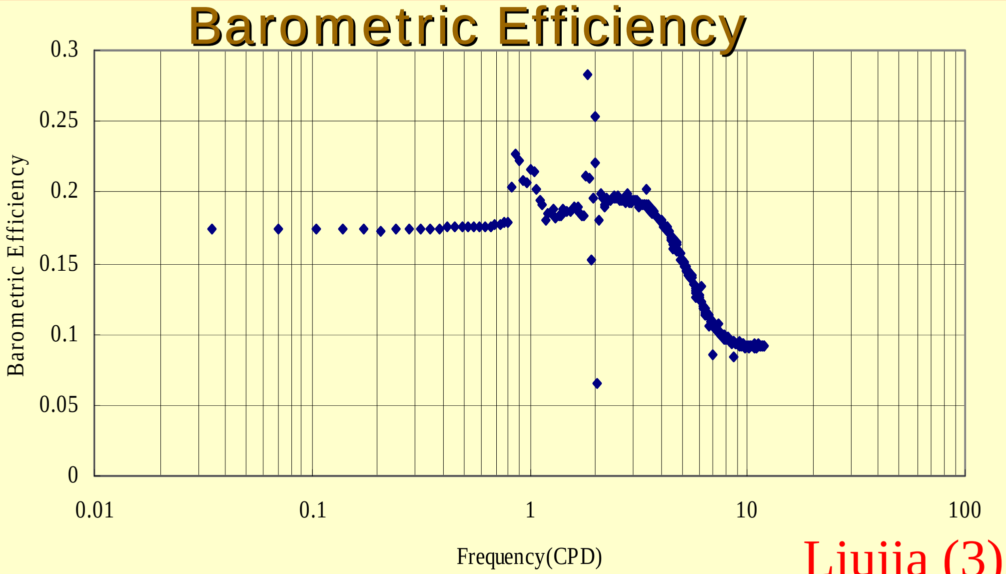


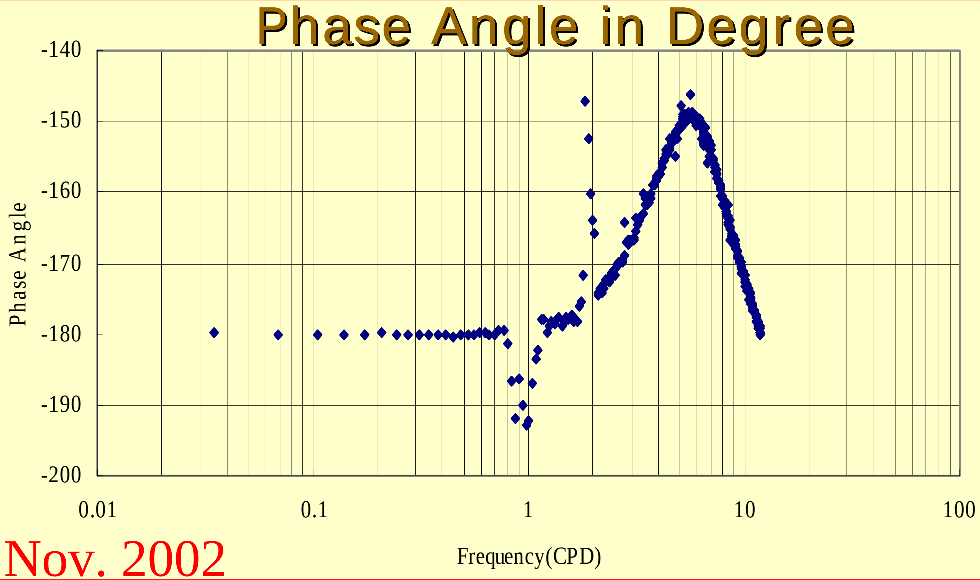
2 Minute Data



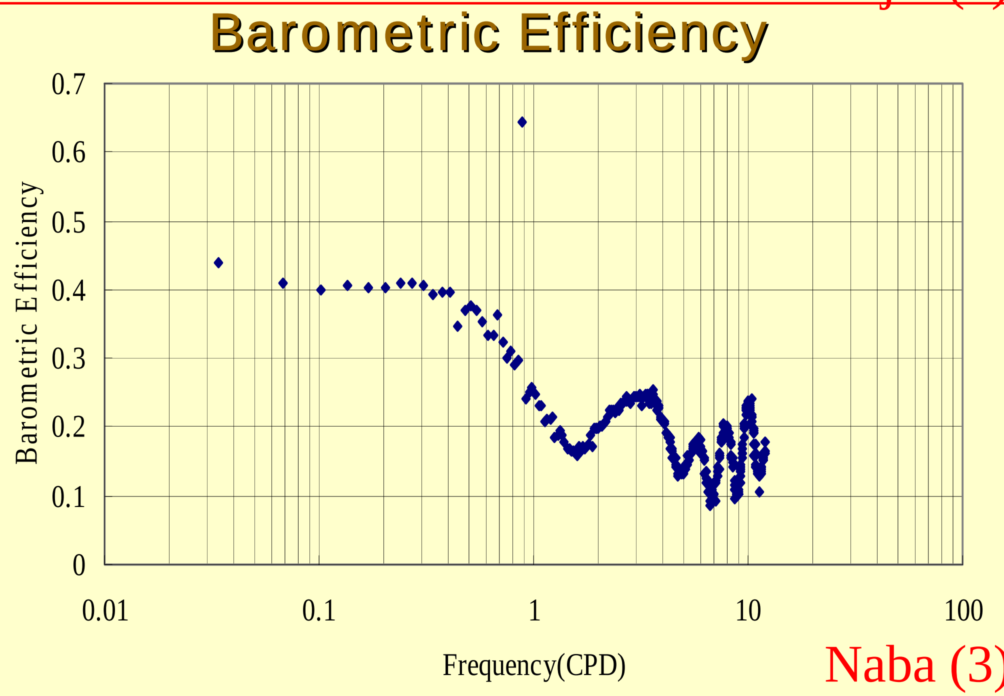


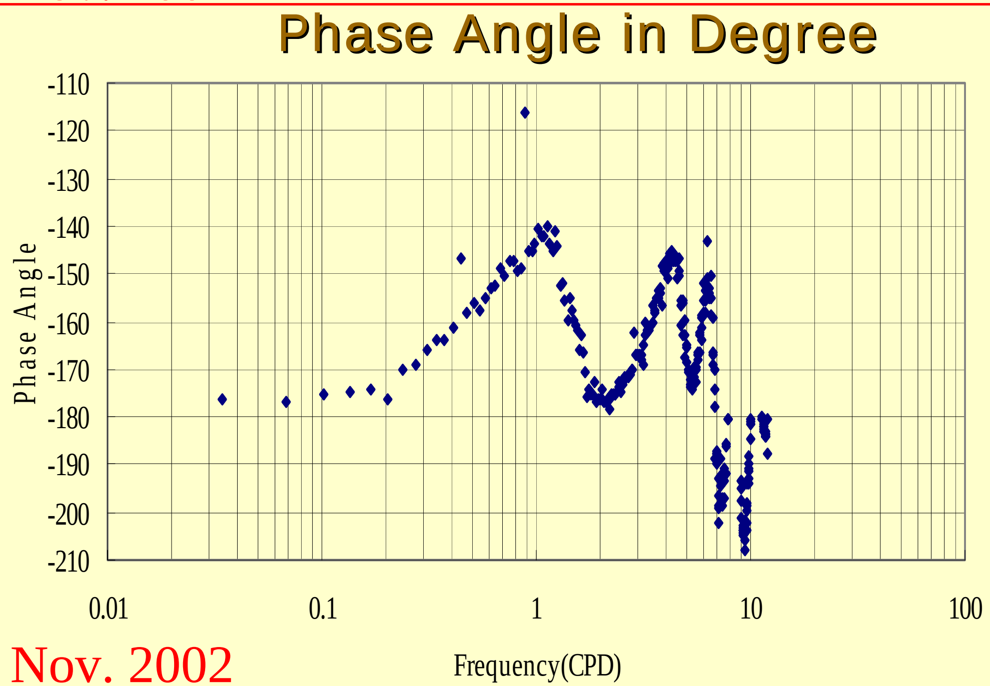
Results After Moving Long-Term Trend (I)





Liujia (3) Nov. 2002

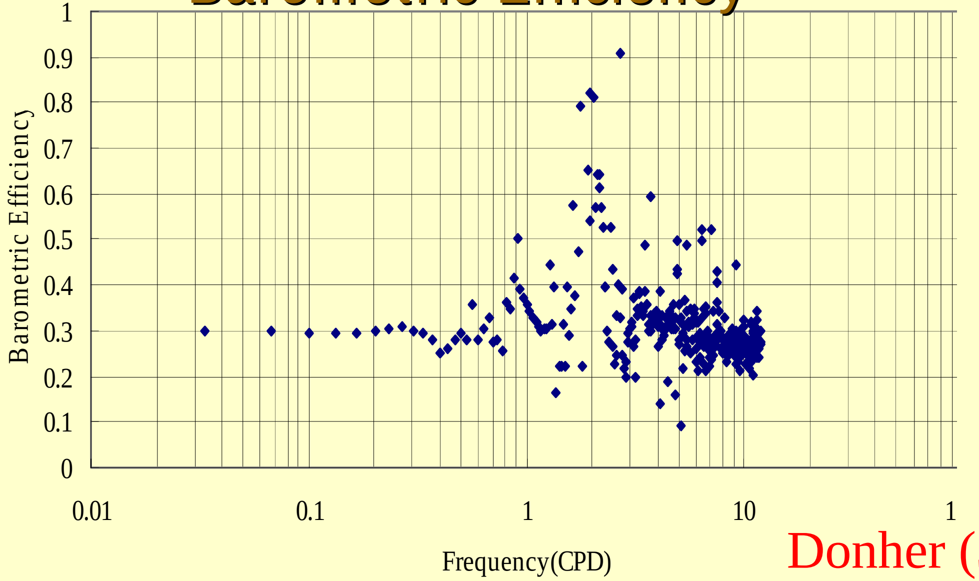




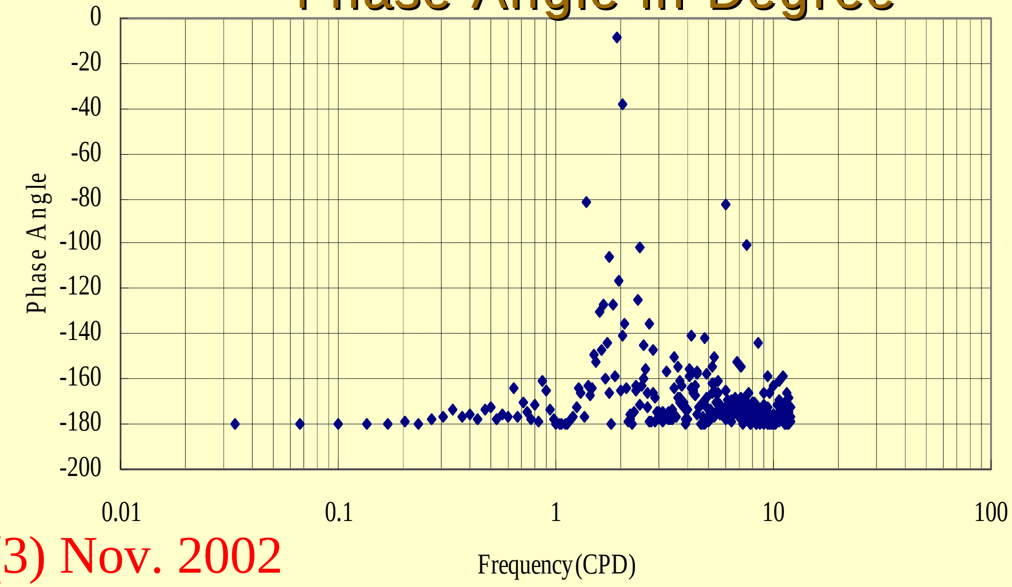
Naba (3) Nov. 2002

Results After Moving Long-Term Trend (II)

Barometric Efficiency

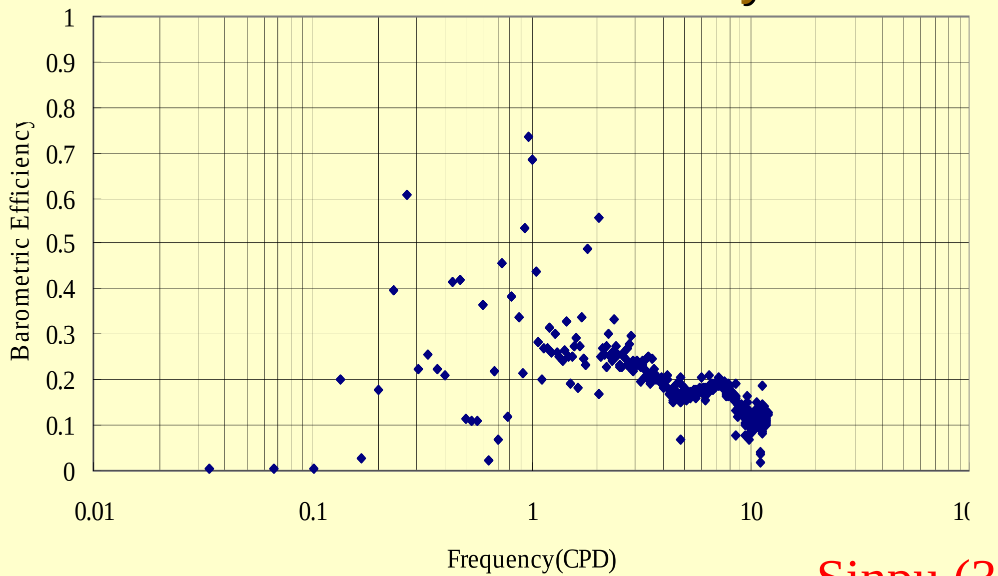


Phase Angle in Degree

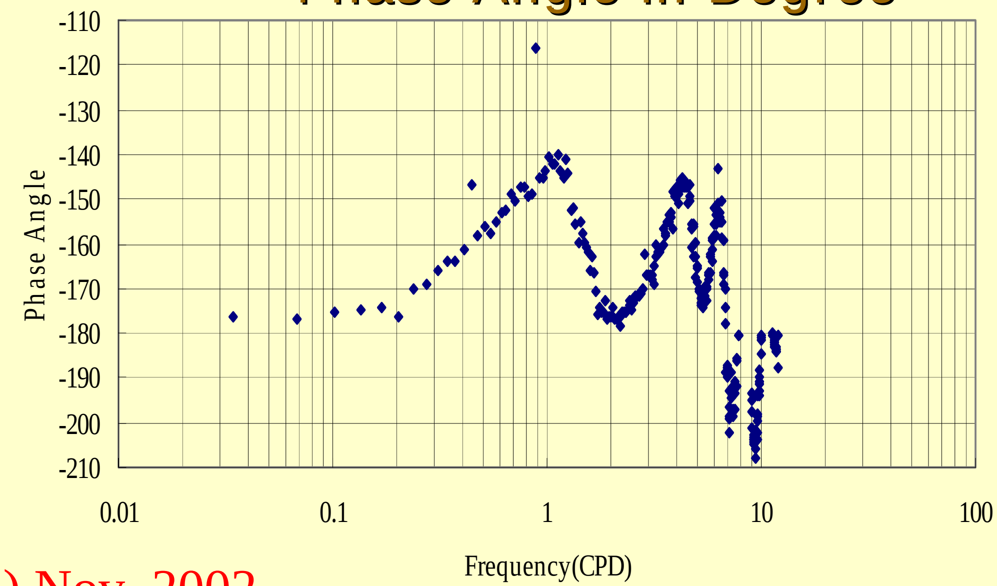


Donher (3) Nov. 2002

Barometric Efficiency



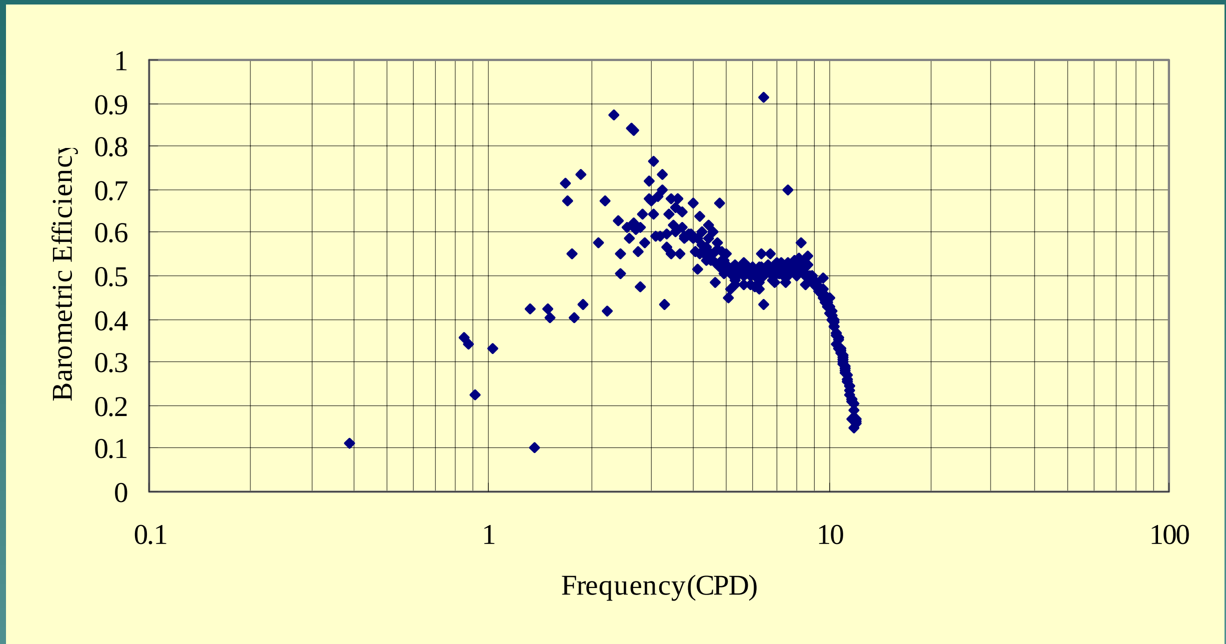
Phase Angle in Degree

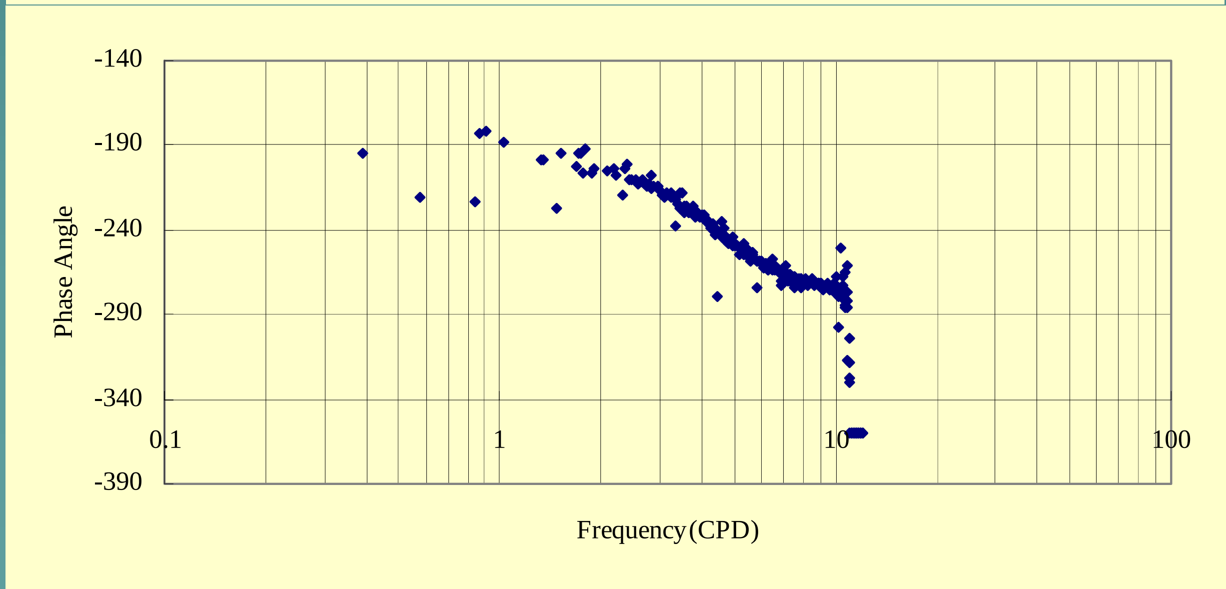


Sinpu (3) Nov. 2002

Results After Moving Long-Term Trend (III)

Tunye (2) Nov. 2002





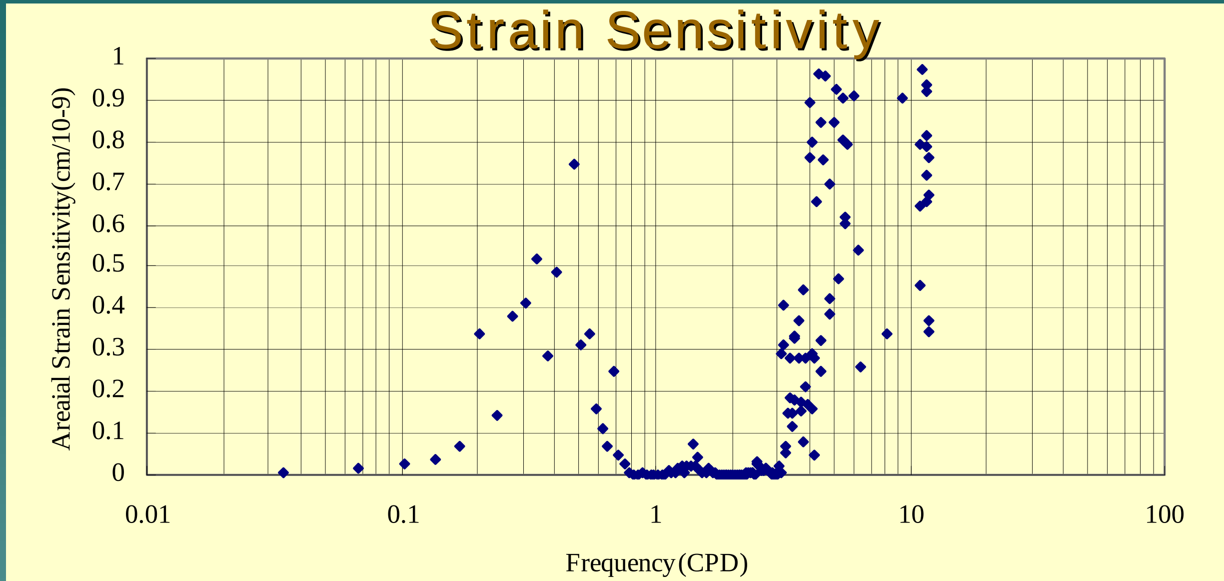
Future plan of the research

- Determination Fluid Flow Properties of Well-Aquifer System

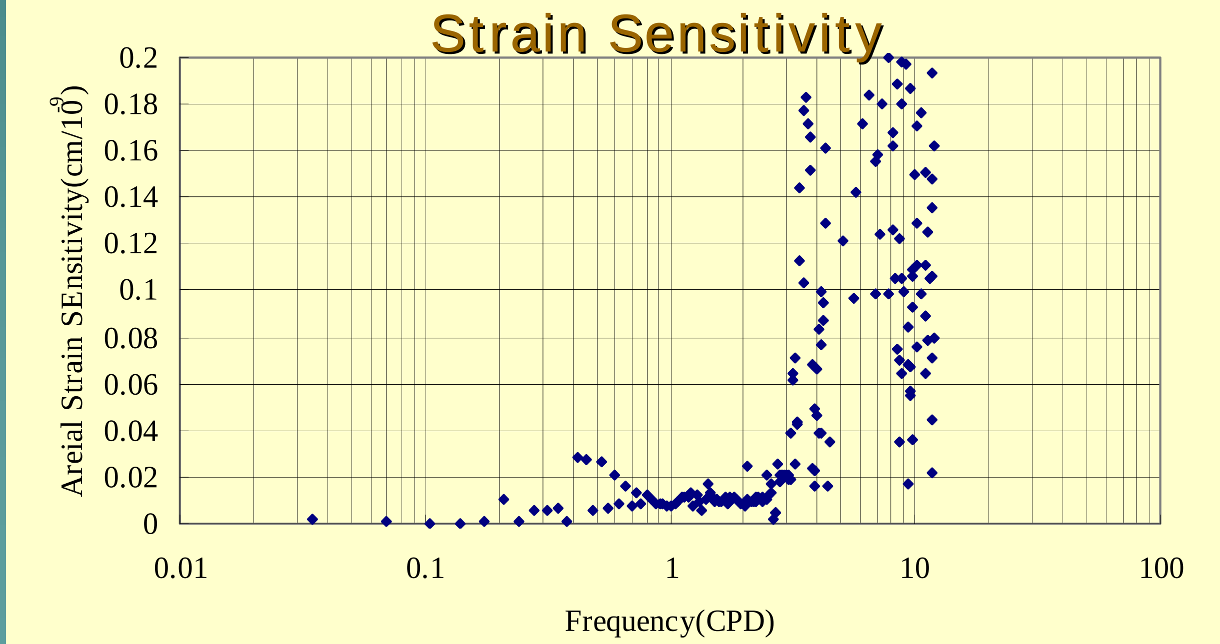
- Transfer Function of Tidal Response

(Response component to volumetric strain)

Transfer Function of Earth Tide (Responses to Areial Strain)



Liujia(3)
Nov. 2002



Naba(3)
Nov. 2002