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# Building a "hydrogeological model" on an "equation friendly platform"

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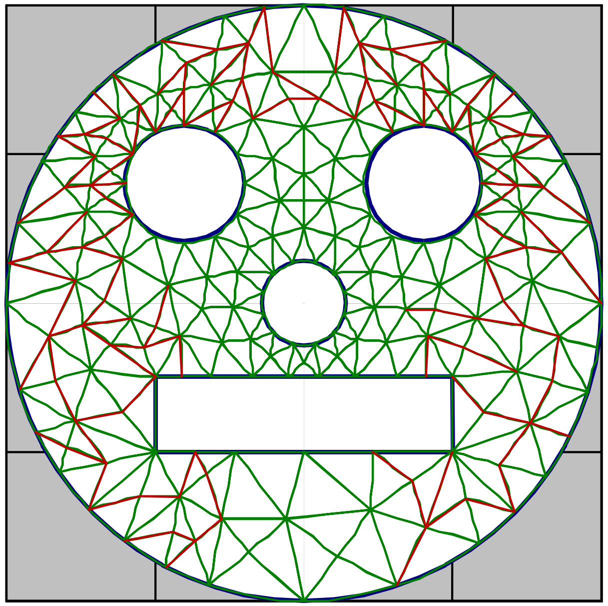
# What do you mean? Equation friendly?

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# Bring on the equations...

- Poroelasticity:
  - Biot(1941)
  - Rice & Cleary(1976)
  - Roeloffs(1996)
- Equations:
  - Law of Geometry
  - Law of Material Constitution
  - Law of Deformation and Flow
- Example:
  - 2D plane strain problems


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# Law of infinitesimal deformation

The total strain - displacement relations in plane strain :

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} = 0 \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix} \begin{Bmatrix} u_x \\ u_y \\ u_z = \text{constant} \end{Bmatrix}$$



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# HILE porous media

For homogeneous isotropic linear elastic porous materials:

The total stress-strain relations in plane strain:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 \\ \nu & 1-\nu & \nu & 0 \\ \nu & \nu & 1-\nu & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz}=0 \\ \gamma_{xy} \end{Bmatrix}$$

The effective stress concept:

$$\begin{Bmatrix} \sigma^e_{xx} \\ \sigma^e_{yy} \\ \sigma^e_{zz} \\ \tau_{xy} \end{Bmatrix} = \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \end{Bmatrix} + \alpha \begin{Bmatrix} p \\ p \\ p \\ 0 \end{Bmatrix}$$

where + stands for tension.

$$\alpha = \frac{3(\nu_u - \nu)}{B(1+\nu_u)}$$



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# Governing law of deformation

The dynamic stress equations:

$$\frac{\partial \sigma^e{}_{xx}}{\partial x} + \frac{\partial \tau^e{}_{xy}}{\partial y} = \rho \frac{\partial^2 u_x}{\partial t^2} + \zeta \frac{\partial u_x}{\partial t}$$

$$\frac{\partial \tau^e{}_{yx}}{\partial x} + \frac{\partial \sigma^e{}_{yy}}{\partial y} = \rho \frac{\partial^2 u_y}{\partial t^2} + \zeta \frac{\partial u_y}{\partial t}$$

**Damping**

**Inertia**

where

$\sigma^e{}_{xx}, \sigma^e{}_{yy}, \tau^e{}_{xy} =$ effective stress components $(\mathrm{Pa} = \mathrm{N/m^2})$,

$(u_x, u_y) =$ displacement vector (m),

$\rho =$ mass density $(\mathrm{kg/m^3})$,

$\zeta =$ damping coefficient $(\mathrm{kg/m^3 s})$,

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# Governing law of flow

The flow equation :

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_{yy}\frac{\partial p}{\partial y}\right)=S_o\frac{\partial p}{\partial t}+\chi\frac{\partial \varepsilon}{\partial t}$$

can be related to strain efficiency

where

$p$ = pore pressure (Pa = N/m$^2$),

$\varepsilon = \frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z}$ = volumetric strain,

$K_{xx}, K_{yy}$ = hydraulic conductivity in x and y directions(m/s),

$S_o$ = storage coefficient(1/m),

$\chi$ = dilation amplifying coefficient(Pa/m).

In classical approach, χ should be related to α, but we treat χ independently.

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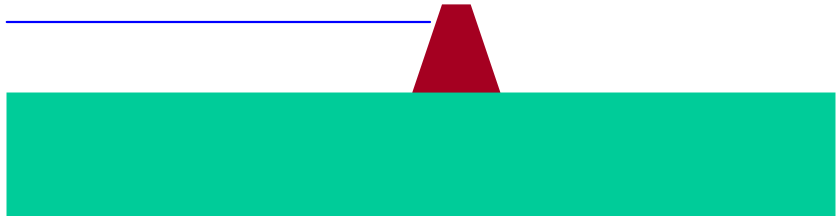
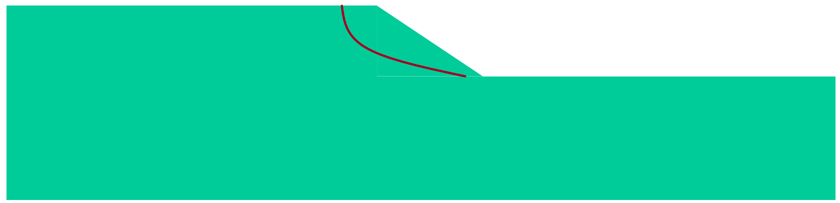
# Simple-minded geotechnical "playgrounds"

- Seepage in dam



- Slope stability



- Foundation analysis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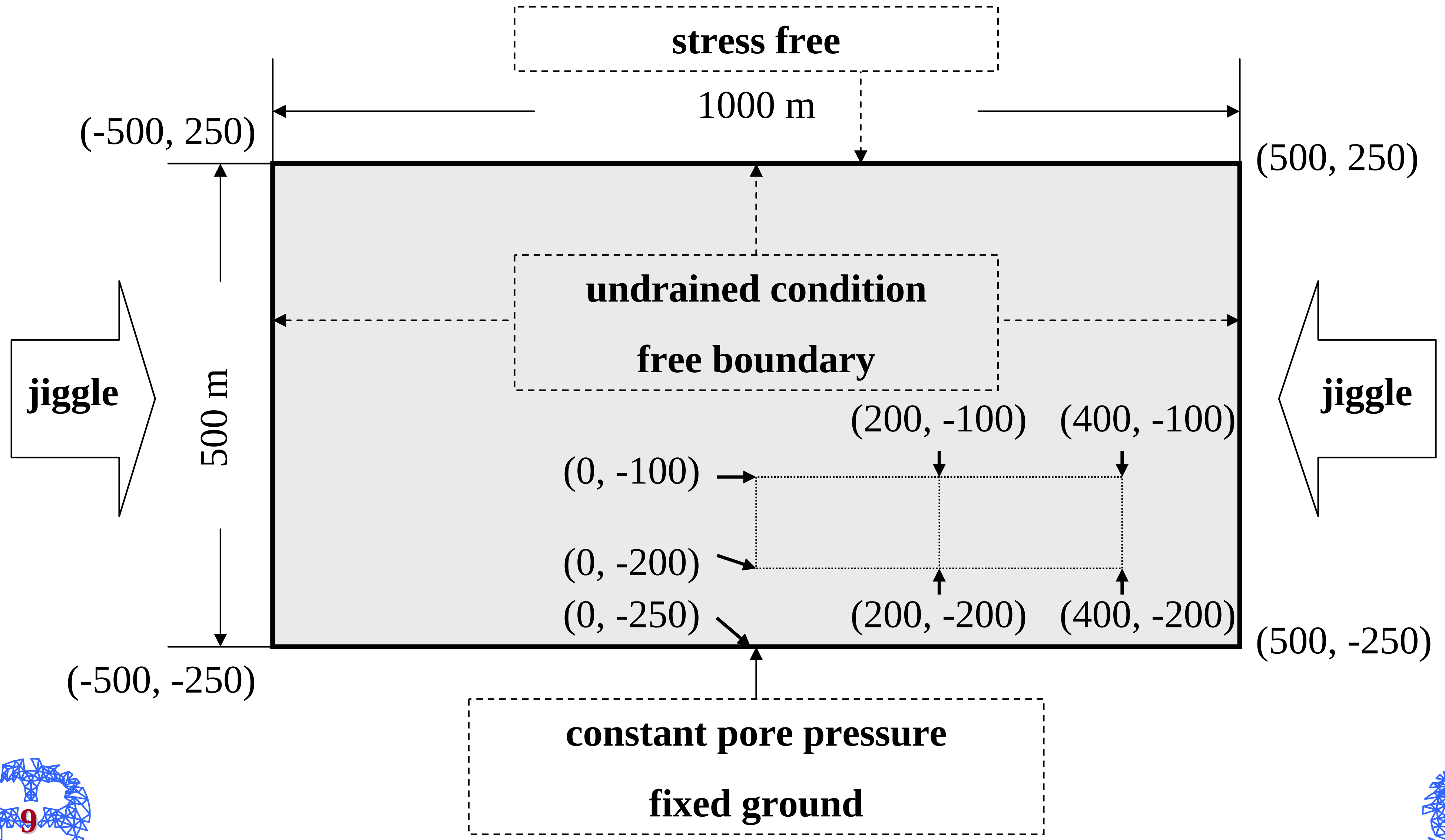
# The problem domain (playground)



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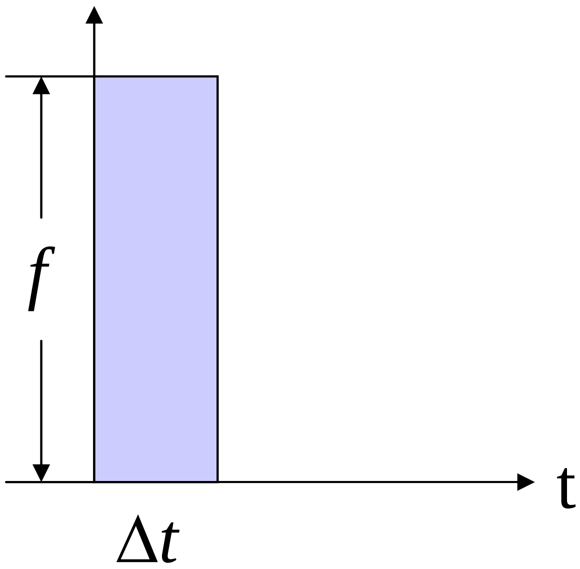
# Jiggle not giggle!

- A “stamp like” function is used to simulate the jiggle driving force.
- The interval, Δt, is chosen to be a small quantity to simulate the impulse-type force.





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# Why jiggle instead of quake?

- Using simple dynamic input helps us grasp the fundamental response spectrum.
- Of course, when the simple case has been said and done, we can go for the more realistic one.


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# Principle of finite element method

- Physical
  - Subdivision and discretization
  - Easy to comprehend and carry out
- Mathematical
  - Variational principle and weak formulation
  - This approach has been so abused that some people even called it a crime!!(Fellipa, 2003)


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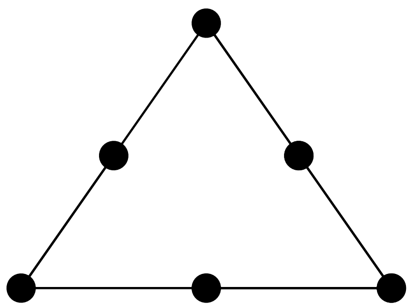
# Choice of finite elements

- For second-order linear problems, the 6-node triangular isoparametric elements offers the most popular choice.
- The accuracy of solution is achieved by a denser grid(meshing) scheme.

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# Equation solving has come a long way . . .

- Must we teach & learn mathematics as in the 1800s?
  - Professor Backstrom, University of Umea, Sweden
- Must we develop the finite element model as in the 1960s?
- New and powerful computer algebra systems(CAS) have become indispensable tools for researchers.


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# Layman's approach: An "equation friendly solver" in need

- I can not be a hydrologist, a geologist, and a finite element modeler at the same time.
- Sure, I would love to be a hydrogeologist. A finite element programmer? Sorry, not interested. Does it take a long time?
- If I develop a hydrogeological model, I would like to solve them in a hurry and spend more time to do what I really want to do: fit this model into the real world!
- A black-box solver is all right, but I would like to have the flexibility of modifying the equations if necessary.
- If I have a box like Pandora's, then I'd be a finite element modeler in no time!


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# Introducing PDEase2D

- *Macsyma*
  - A Computer Algebra System(CAS).
  - Developed at Massachusetts Institute of Technology as early as 1982.
  - One of earliest symbolic analysis software.
- *PDEase2D*
  - *P*artial *D*ifferential *E*quation solver made *ease* for *2D* IVP/BVP problems.
  - Actually, this solver helps me avoid “low-level” programming.
- *PDEase2D 2.2* (1997)
  - This version is out of the market but still runs.
  - Current version is called *FlexPDE*.
  - A *MATLAB* toolbox, *FEMLAB*, does the same trick.


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# Principal Features of PDEase2D

1. Language-Based Problem Specification.
2. Galerkin Finite Element Spatial Dependence Solver.
3. Automated Adaptive Grid Refinement.
4. Evolution Time Dependence Solver.
5. Automated Time Step Refinement.
6. Essential and Natural Boundary Conditions.
7. Multi-Region Problems.
8. Eigenvalue Modal Analysis.
9. Non-Analytical Data Import and Export.
10. Automatic or User-Controlled Solution Flow.
11. Graphic Output and Animation.
12. DXF Format Support.


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# Using PDEase2D

1. Create a mathematical model of the system.
2. Translate the model into a problem descriptor.
3. Open Macsyma Front End to run the program.
4. Review, animate, and print the solution.


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# Meshing convergence (Grid control in PDEase2D)

| Grid control | Nodes | Cells(Elements) | Run time ( Real time = 1 sec ) | Pore pressure ratio (400,-100) at t = 1 sec | Convergence (Compare to 5153-node sol.) |
|---|---|---|---|---|---|
| 21 | 867 | 402 | 2’17“ | 0.000971485 | 0.18 |
| **31** | **1951** | **928** | **6‘6“** | **0.001102204** | **0.00977** |
| 41 | 3401 | 1638 | 12’48“ | 0.001187725 | 0.00729 |
| 51 | 5153 | 2500 | 19’55“ | **0.001196451** | 0 |

**Bench Testing Platform:**

**Hardware: AMD Athlon 1.2 GHz, SDRAM 100 MHz**

**Software: Windows 2000, PDEase2D 2.2**

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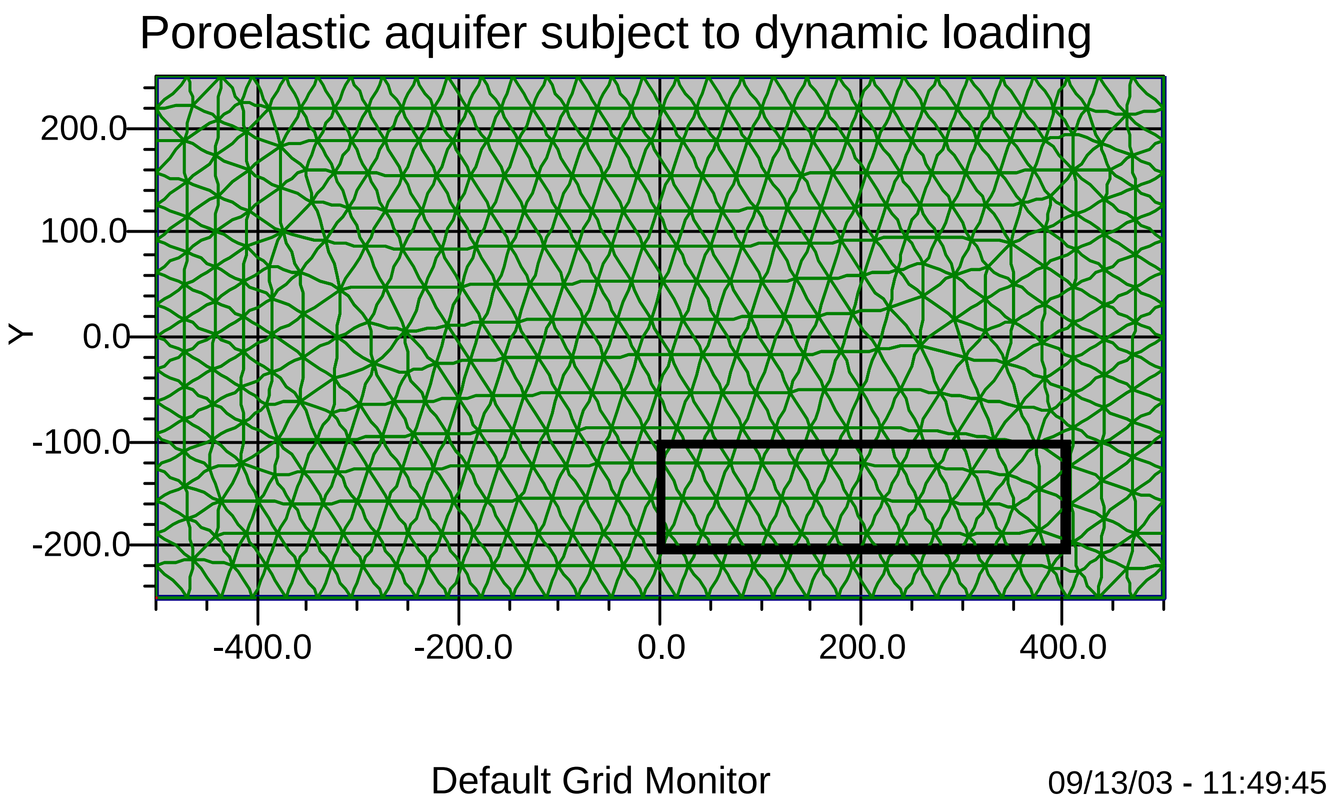
# Nodes = 1951, Cells = 928





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# Assessing numerical stability

- Running the commercial program with its default always asks for error.
- Make a careful choice of time stepping methods:
  - THETA = 0 Euler
  - THETA = 1/2 Crank-Nicolson (default)
  - THETA = 2/3 Galerkin
  - THETA = 1 Backward difference


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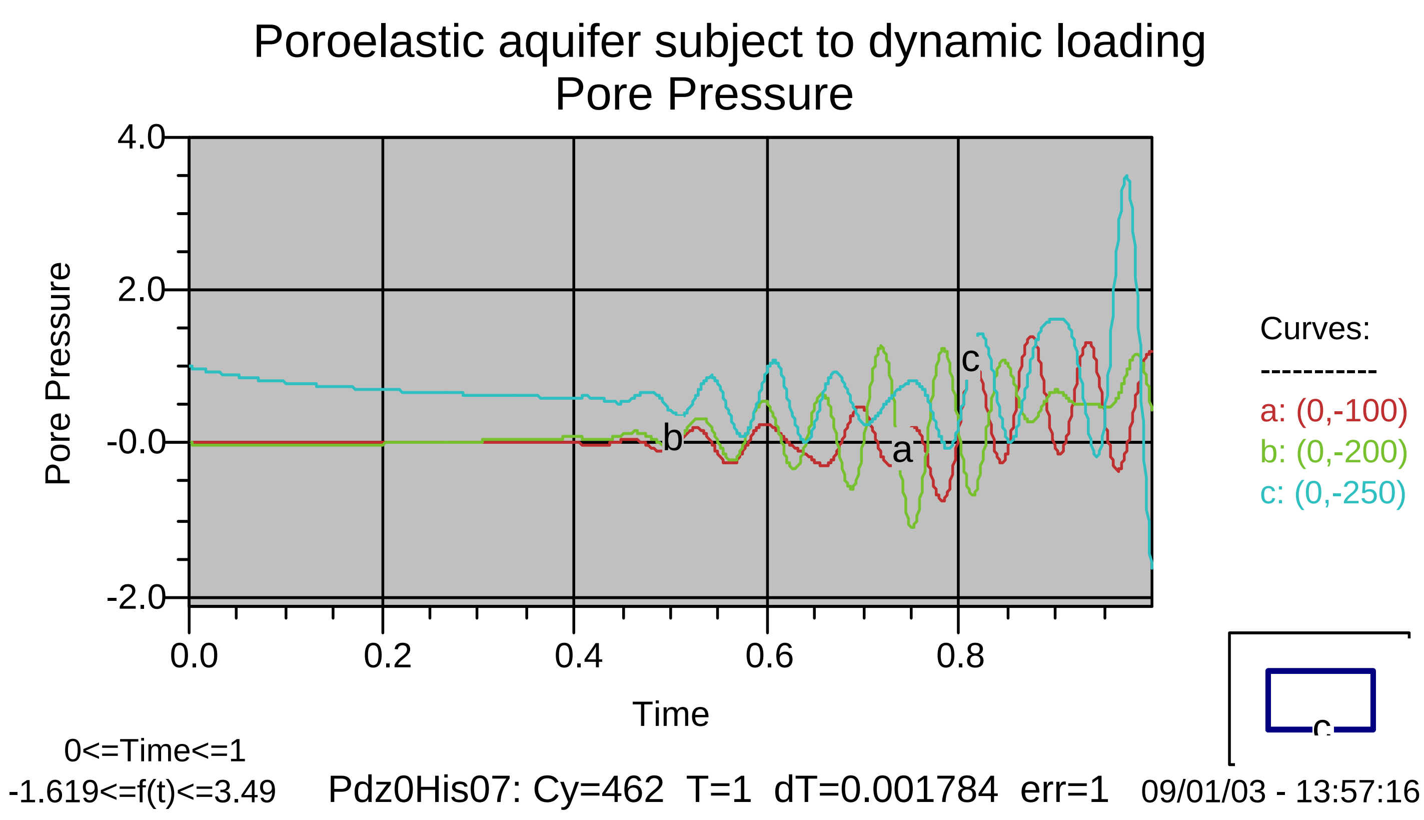
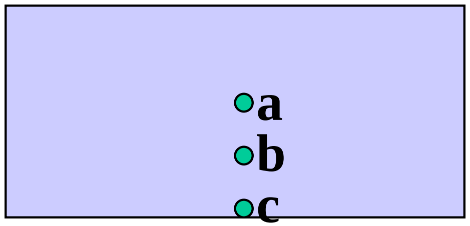
# Euler

# (THETA = 0)





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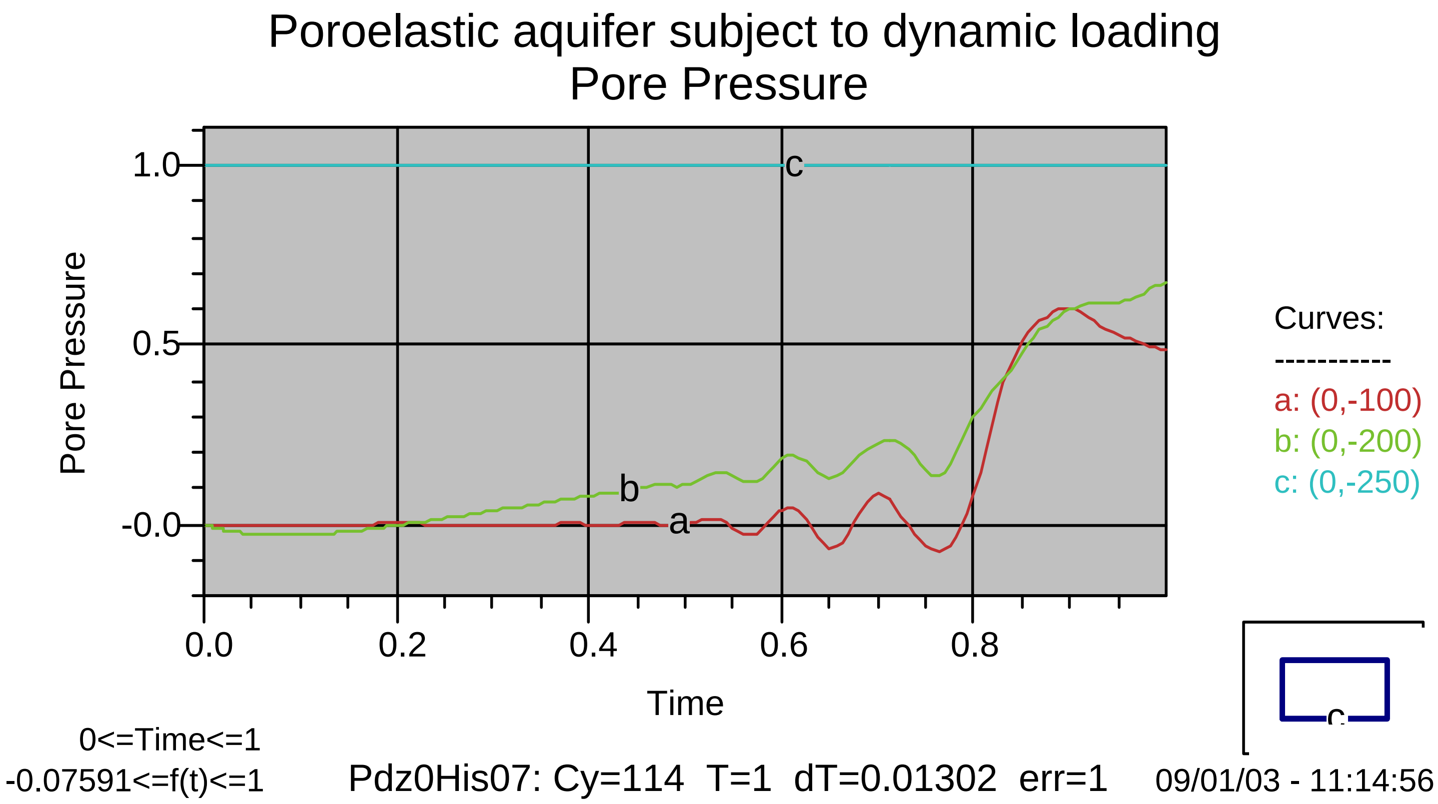
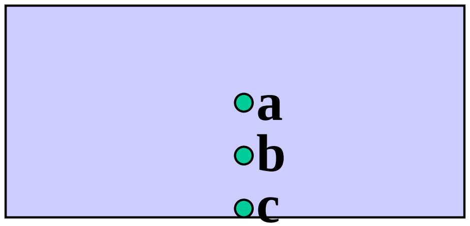
# Crank-Nicolson

# (THETA = 1/2)



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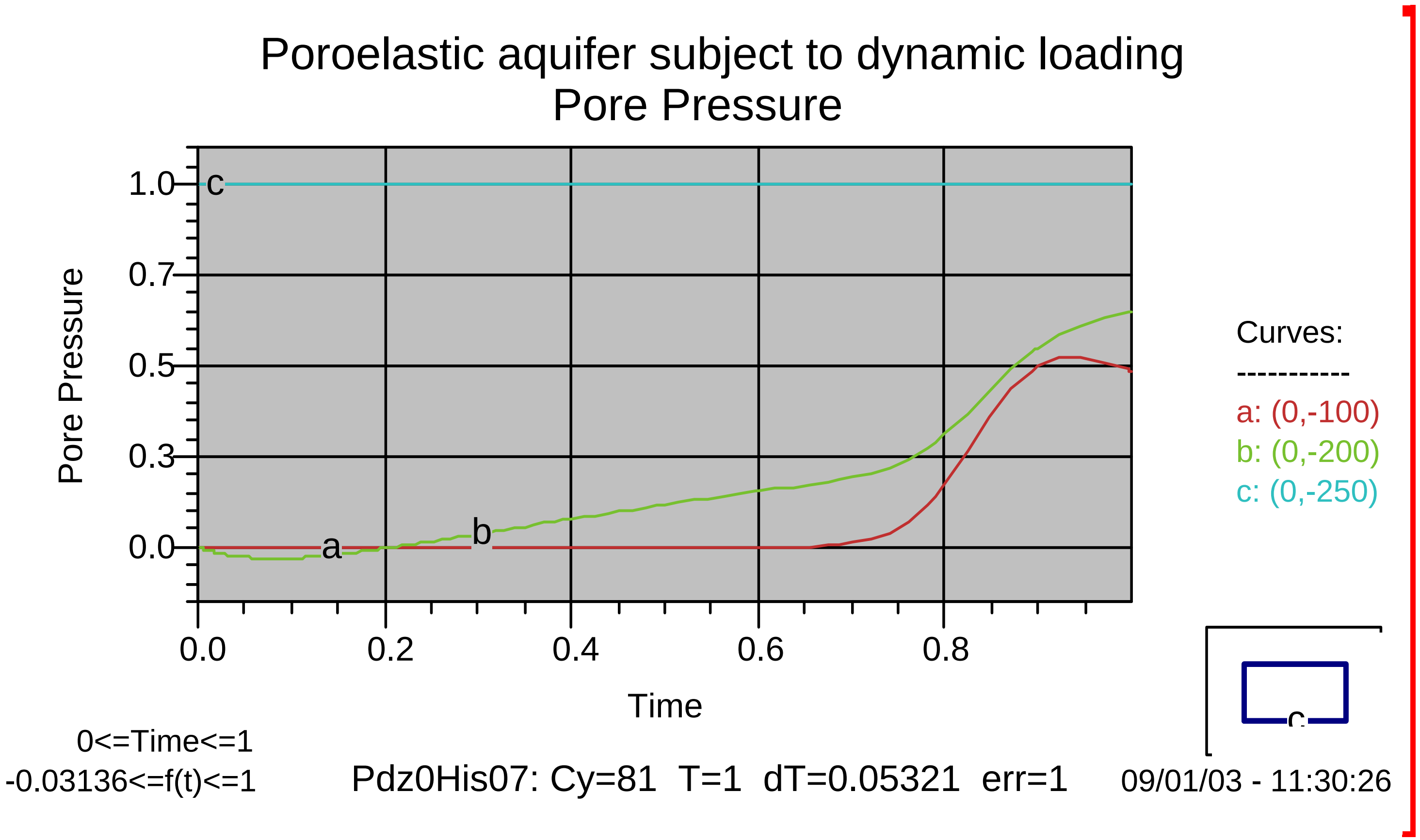
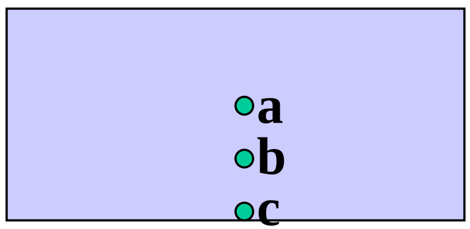
# Galerkin

# (THETA = 2/3)



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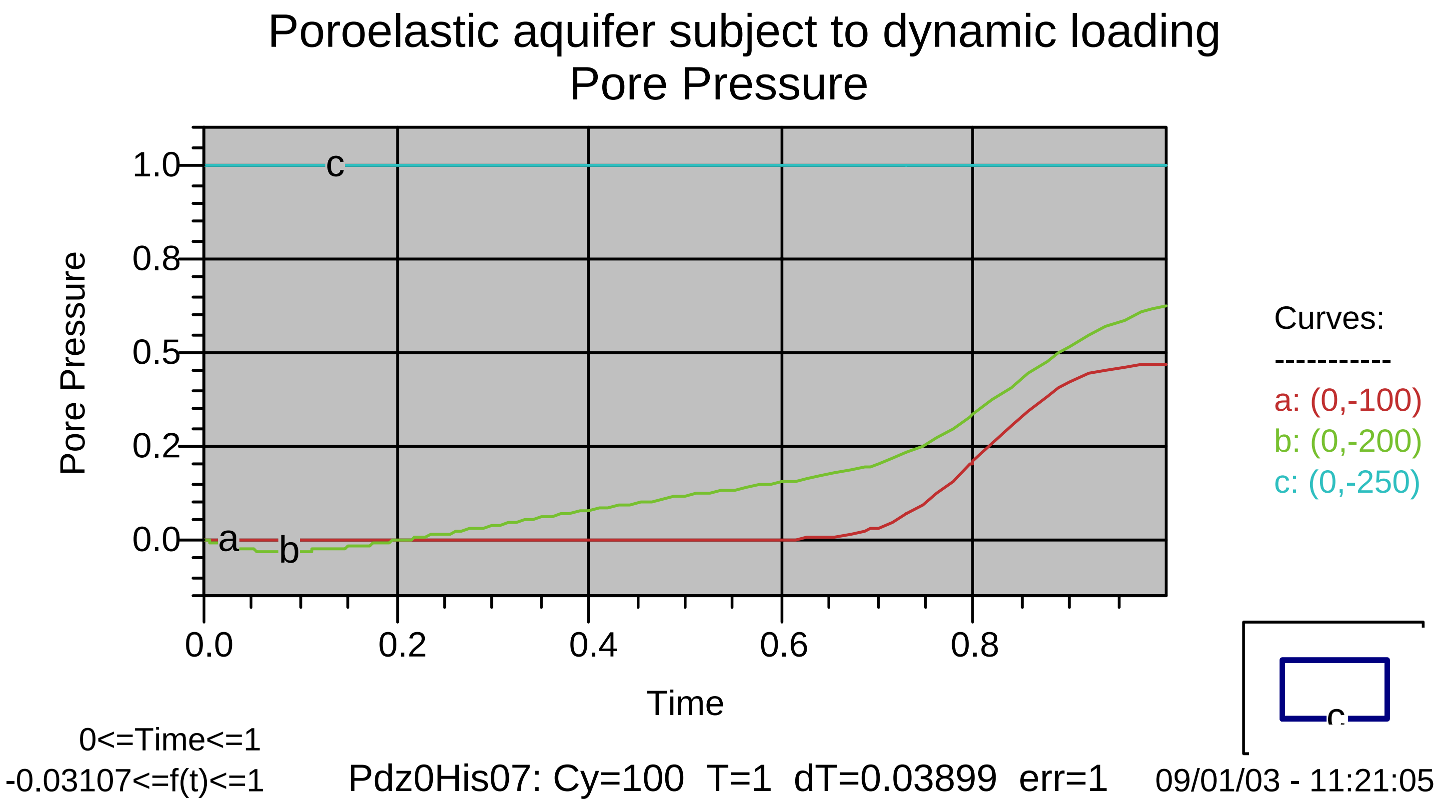
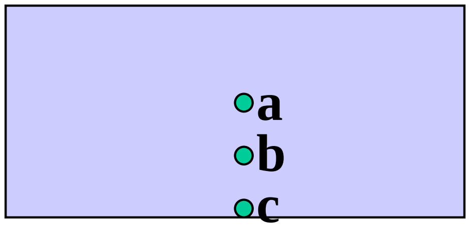
# Backward Difference

# (THETA = 1)



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# Marching time backward!

- Both Euler and Crank-Nicolson methods produce oscillations due to numerical instability.
- Both Galerkin weight and backward difference methods are able to suppress the undesirable ripple.
- This study adapts backward difference as the time stepping scheme.


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# Sensitivity study

- Study the effects of various parameters on different locations of the playground:
  - jiggle
  - storage coefficient
  - dilation amplifying coefficient

- $$\text{Pore pressure ratio} = \frac{\text{Pore pressure at } (X, Y)}{\text{Pore pressure at bottom boundary}}$$


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# Input parameters

| | |
|---|---|
| Mass density : | $\rho = 2700 \text{ kg/m}^3$ |
| Young's modulus : | $E = 10^{10} \text{ Pa}$ |
| Poisson's ratio : | $\nu = 0.3$ |
| Skempton's coefficient : | $B = 0.111$ |
| Undrained Poisson's ratio : | $\nu_u = 0.35$ |
| Damping coefficient : | $\zeta = 10^4 \text{ kg/m}^3\text{s}$ |
| Hydraulic conductivities : | $K_{xx} = K_{yy} = 1 \text{ m/s}$ |
| Storage coefficient : | $S_o = 10^{-3} \sim 10^{-5} \text{m}^{-1}$ |
| Amplifying coefficient : | $\chi = 0 \sim 10^3 \text{Pa/m}$ |



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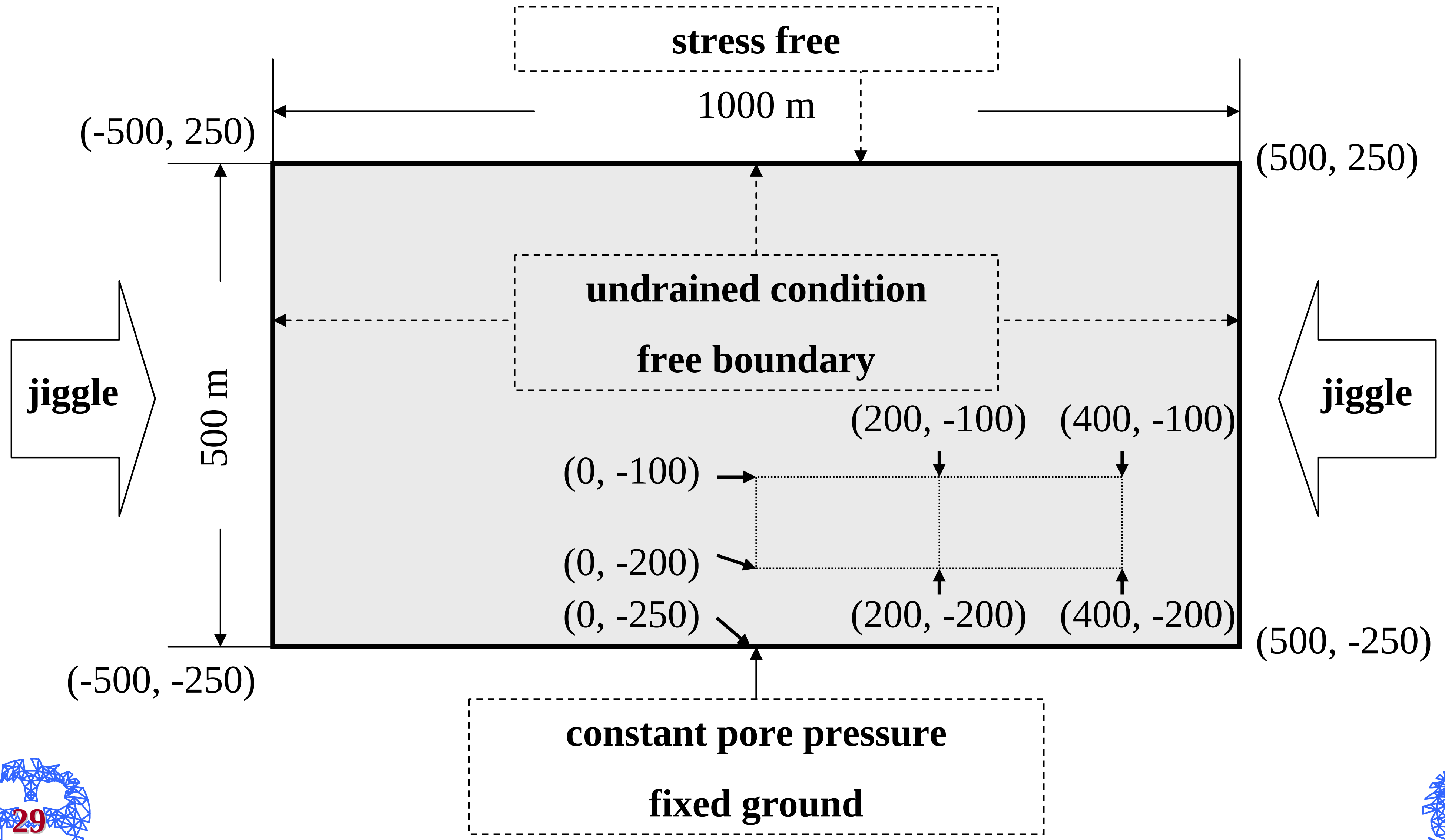
# Playground



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# Visualization

- Displacement vector plot
- Pore pressure distribution
- Shear stress distribution
- etc . . .


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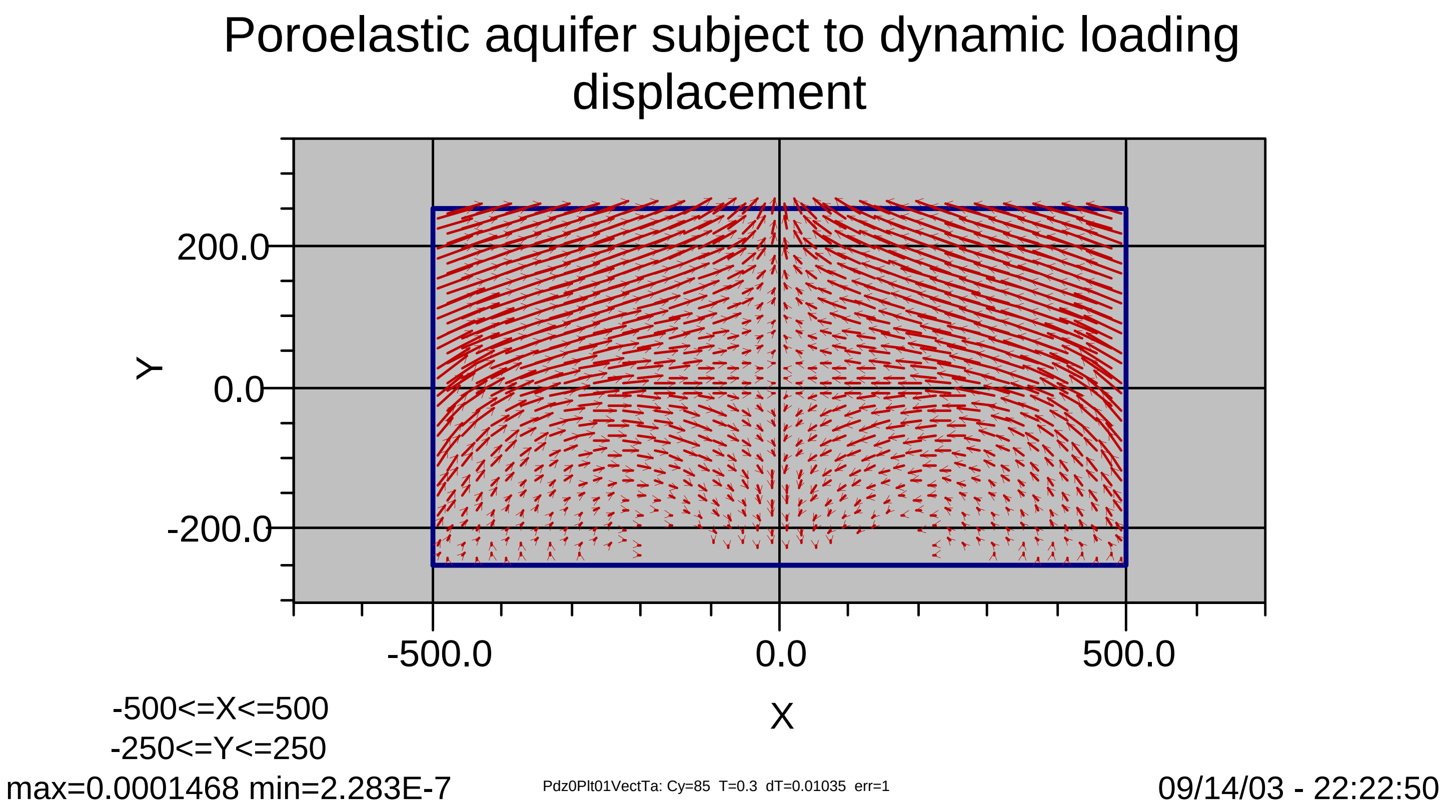
# Displacement field




Displacement field at t = 0.3 sec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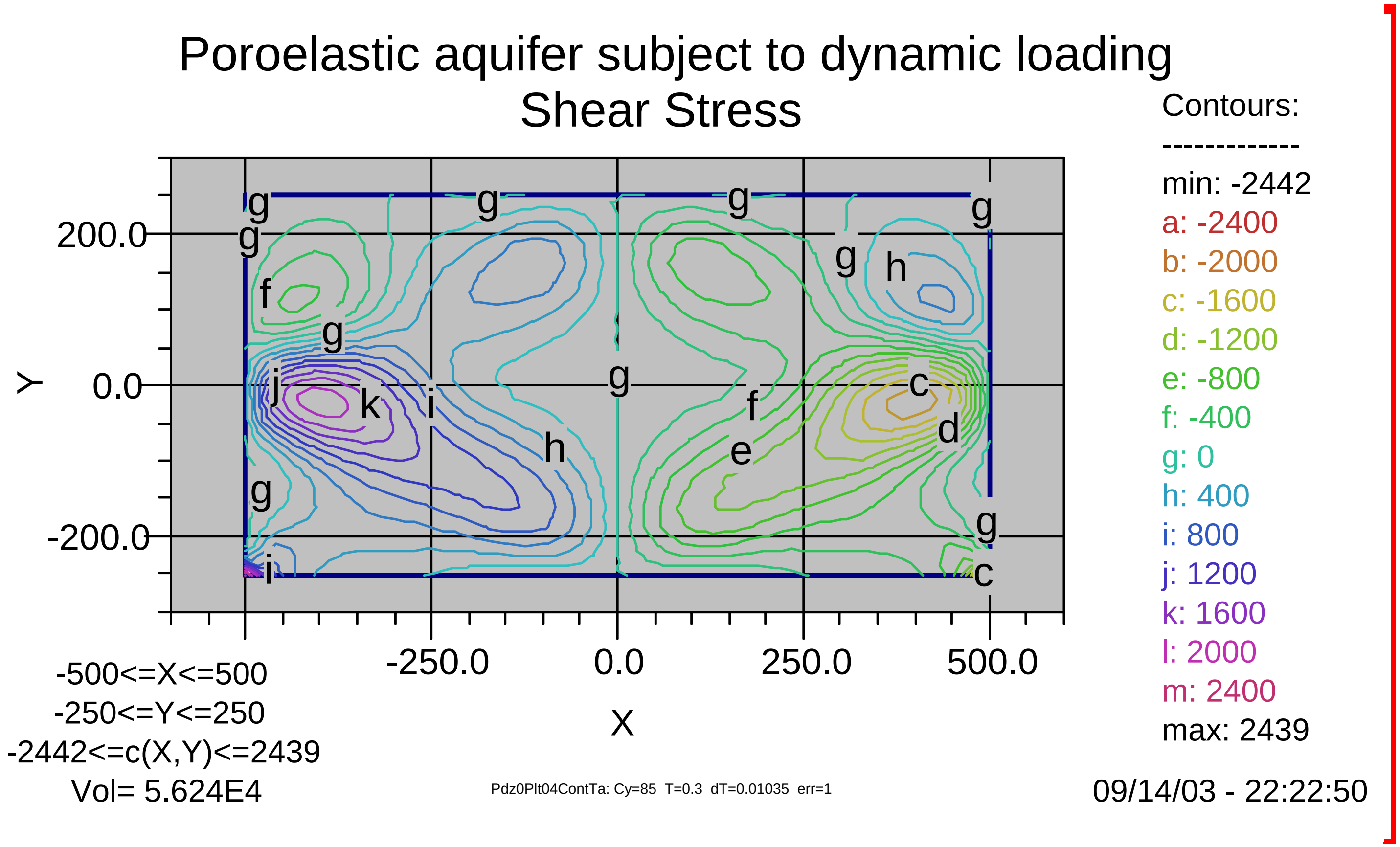
# Shear stress contour




Shear stress at t = 0.3 sec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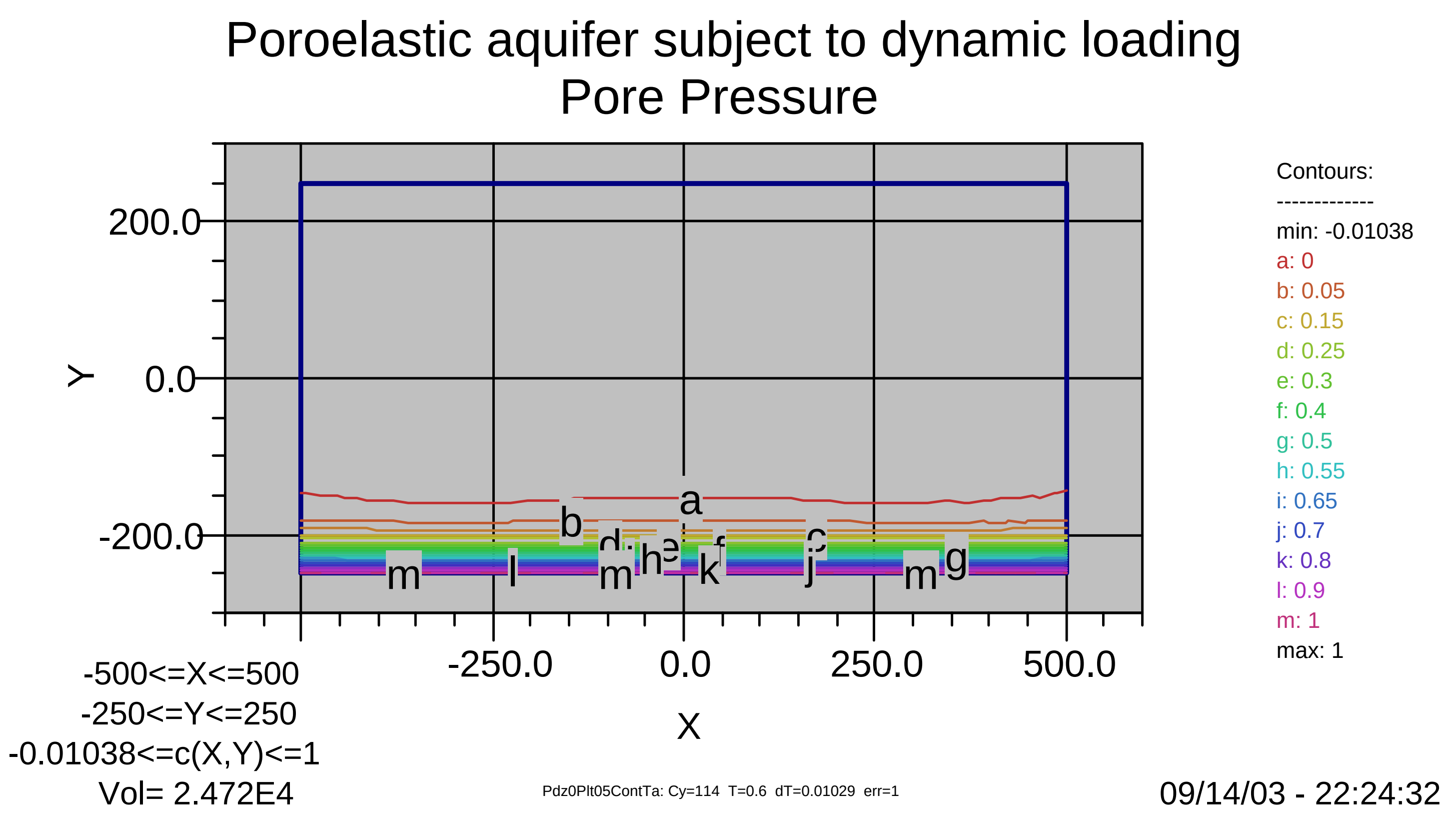
# Pore pressure contour




Pore pressure at t = 0.3 sec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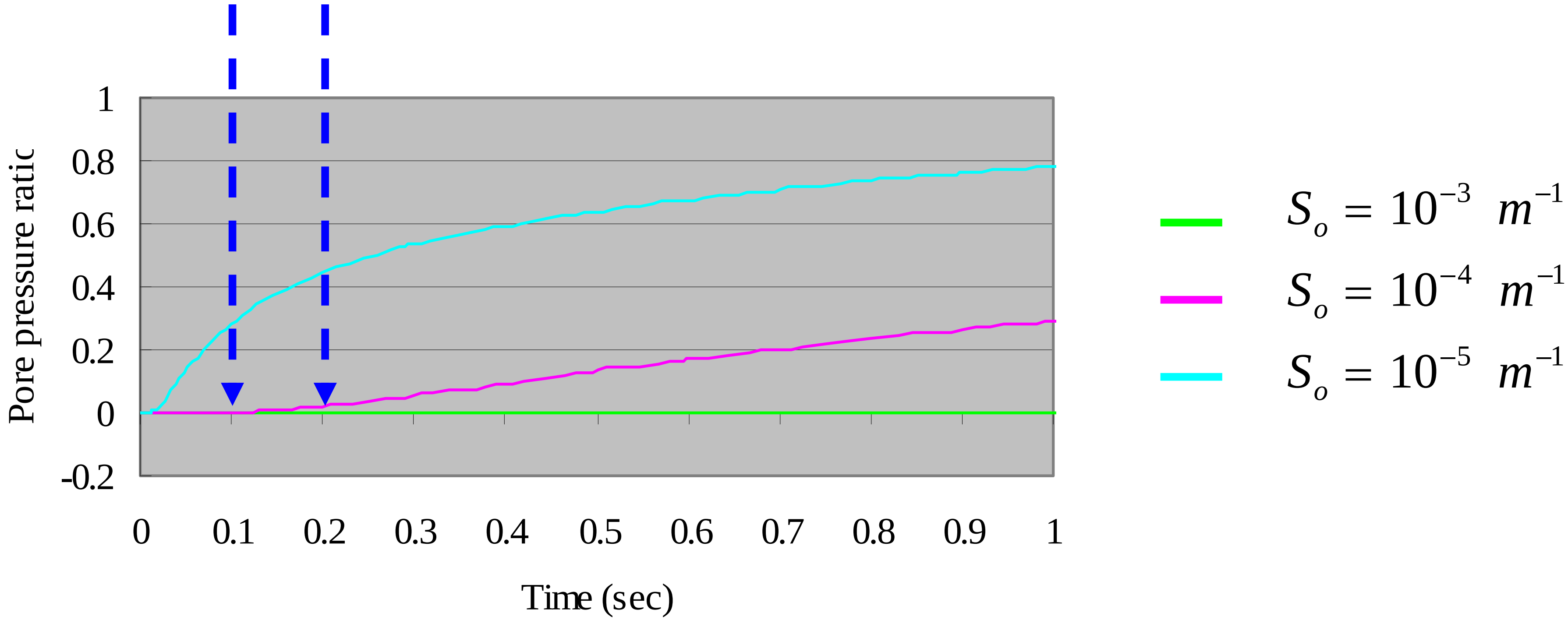
# Storage coefficient

Pore pressure is monitored at (400,-100)

Jiggle is applied between 0.1 sec < t < 0.2 sec





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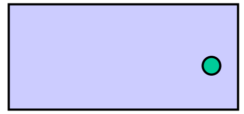
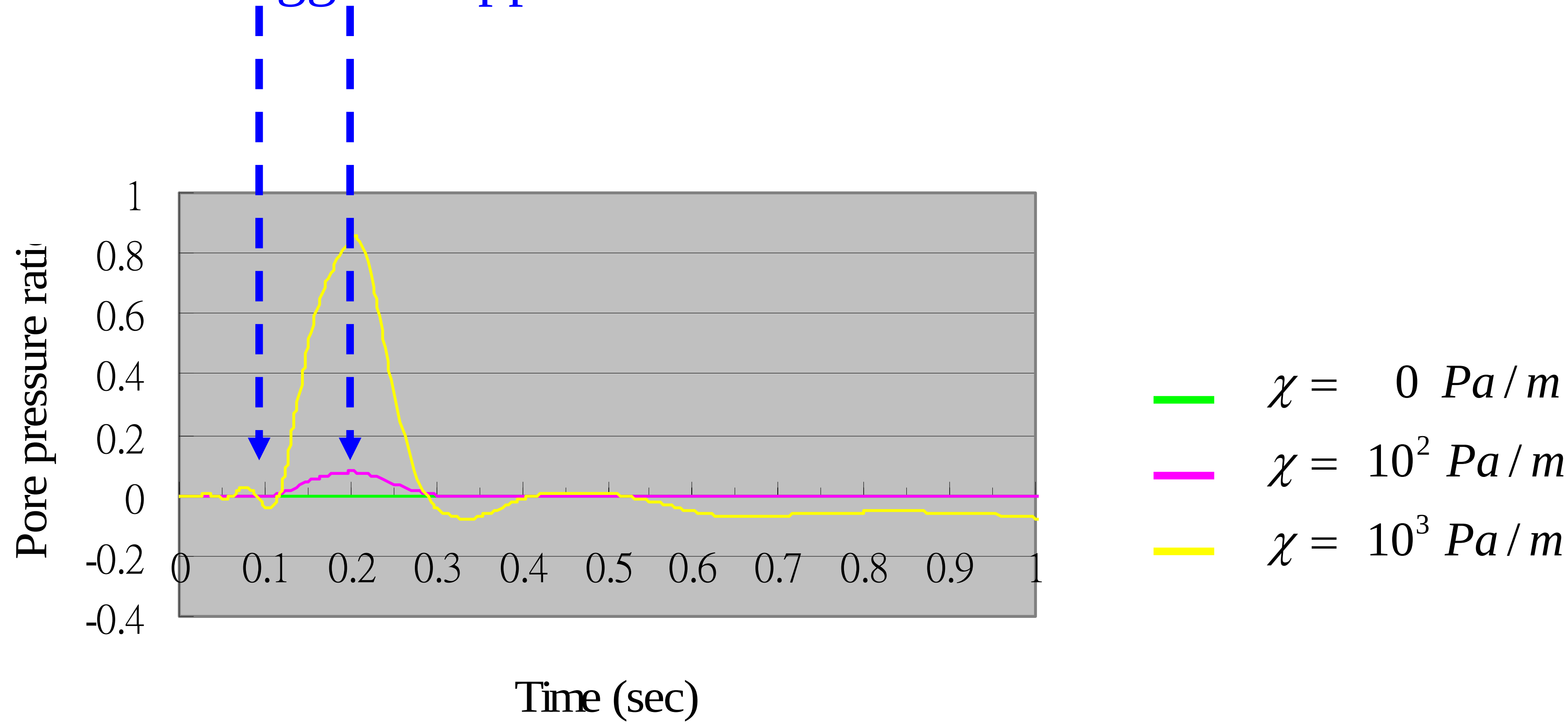
# Dilation amplifying coefficient

Pore pressure is monitored at (400,-100)



Jiggle is applied between 0.1 sec < t < 0.2 sec





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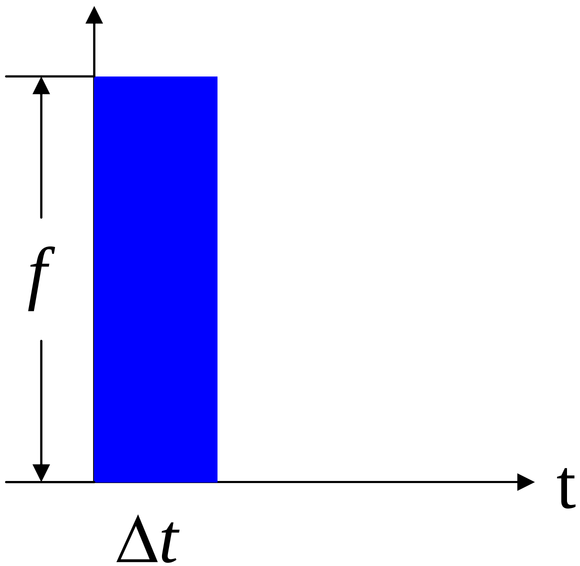
# A stamp is coming!

- The magnitude, *f*, is chosen for the sensitivity study.





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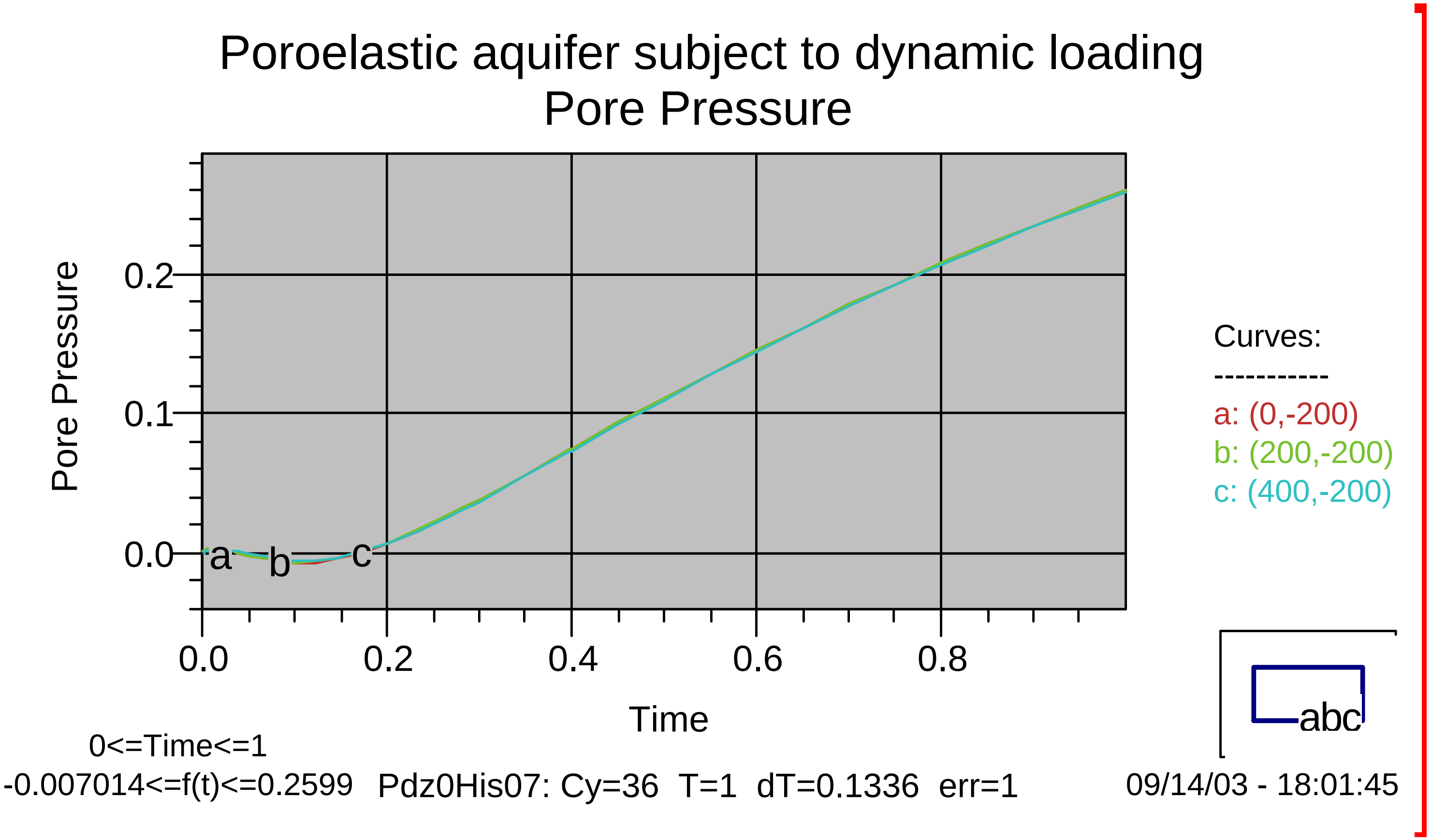
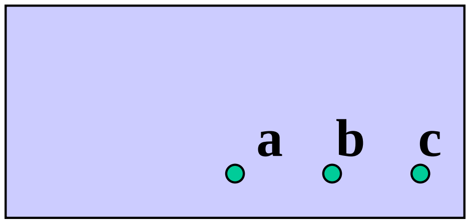
# No stamp:



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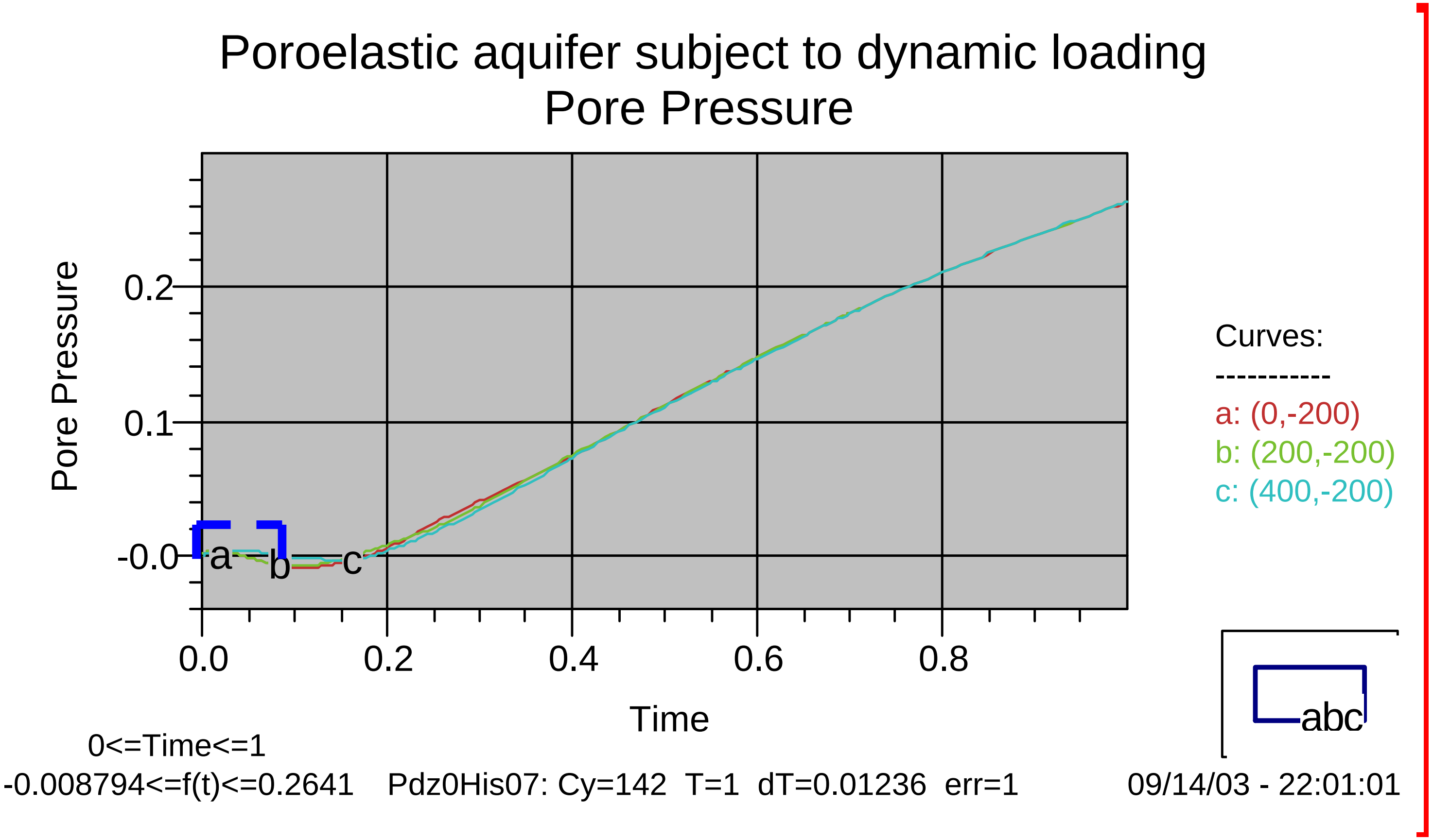
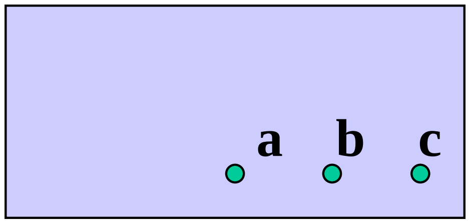
# A small stamp:

## $f = 10^3$ N, $\Delta t = 0.1$ s, $\chi = 100$





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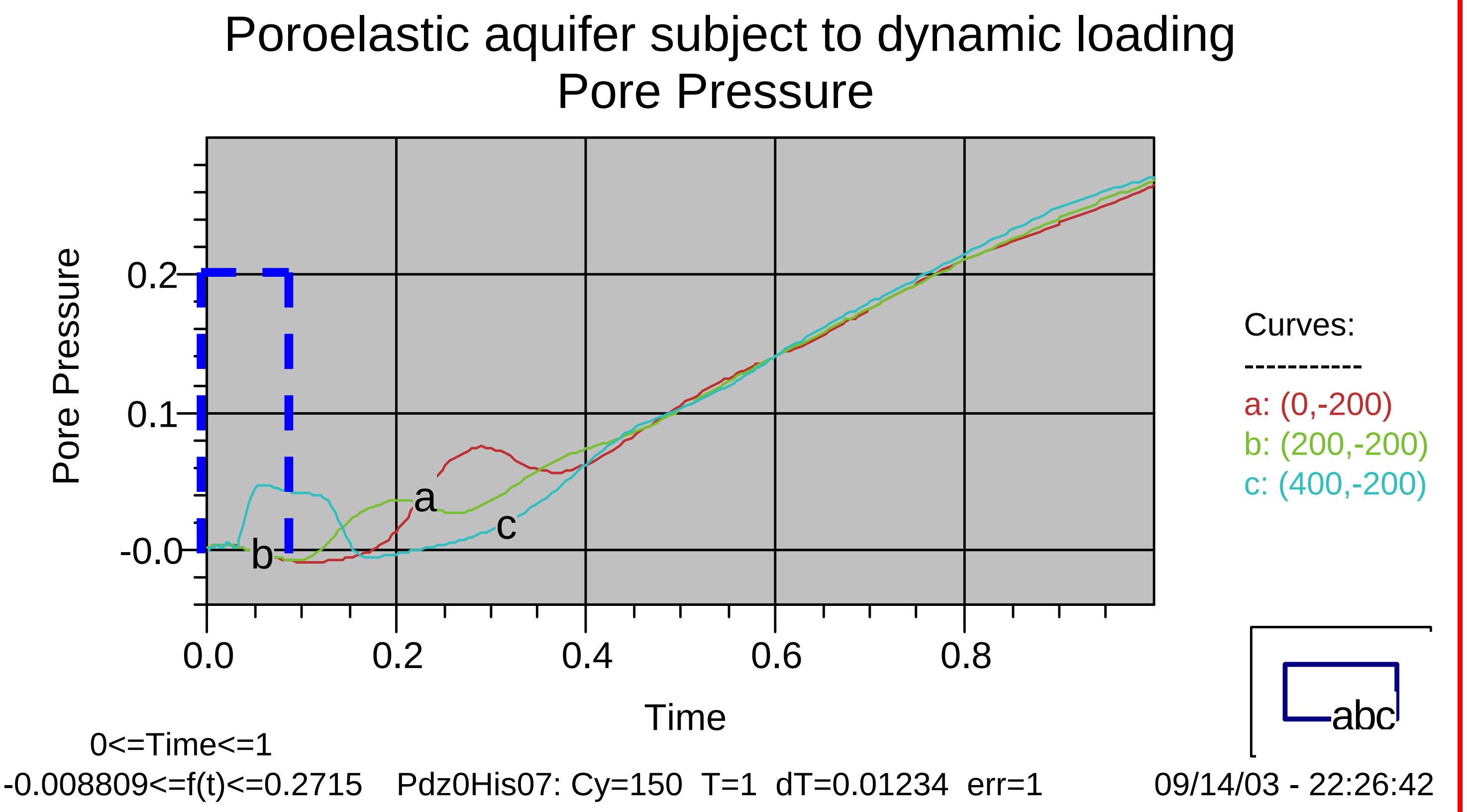
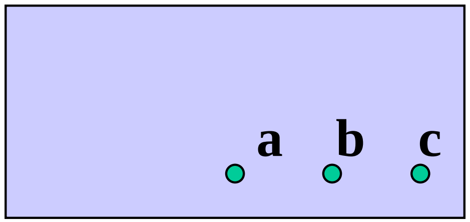
# A big stamp:

## $f = 10^4$ N, $\Delta t$= 0.1 s, $\chi$ = 100





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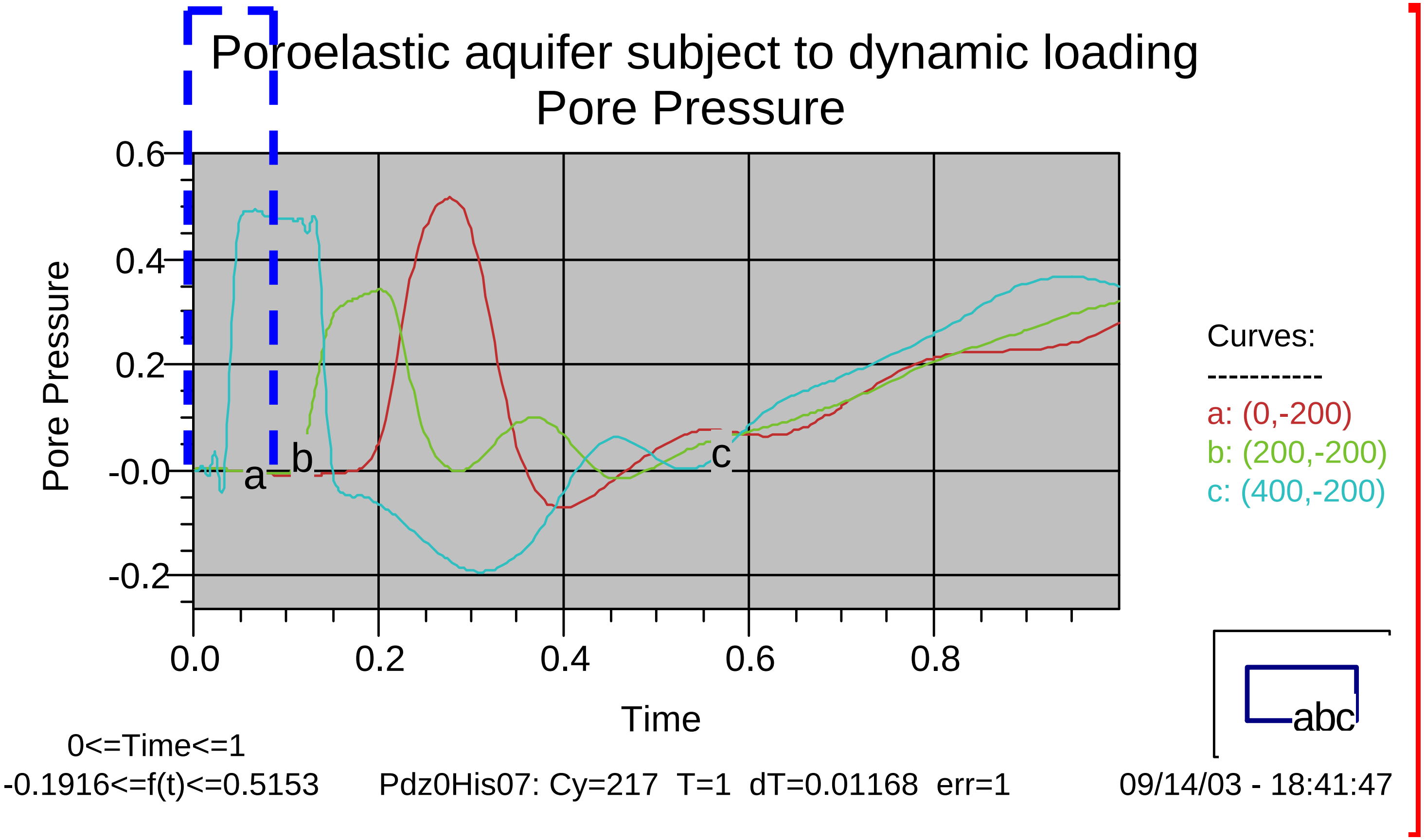
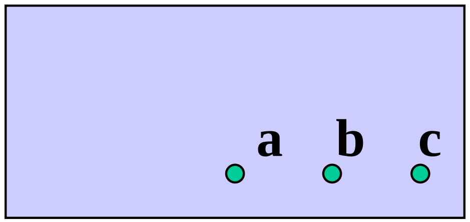
# A bigger stamp:

$$f = 10^5 \text{ N}, \Delta t = 0.1 \text{ s}, \chi = 100$$



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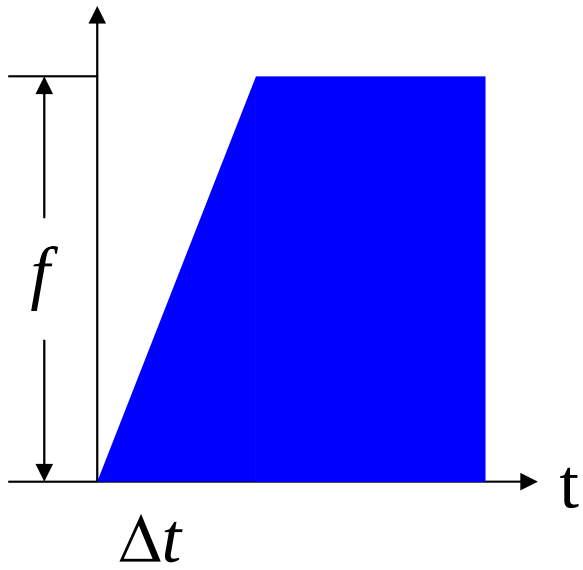
# Now here comes a ramp!

- A “ramp” function is used to simulate the jiggle driving force.





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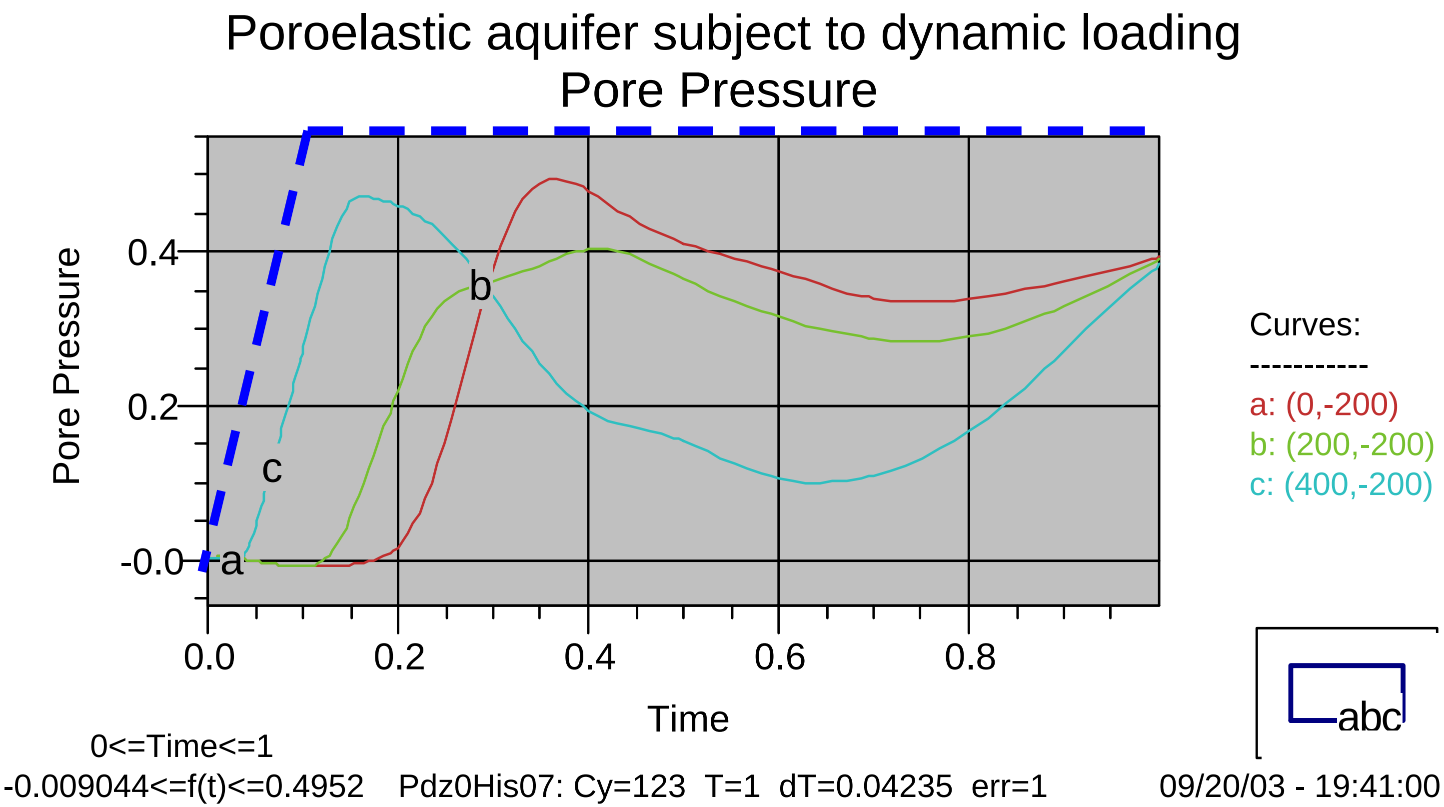
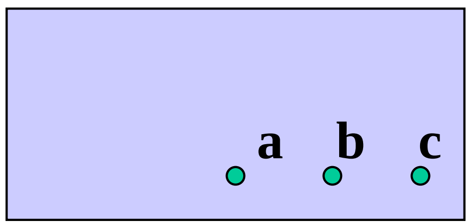
# A “steep” ramp:

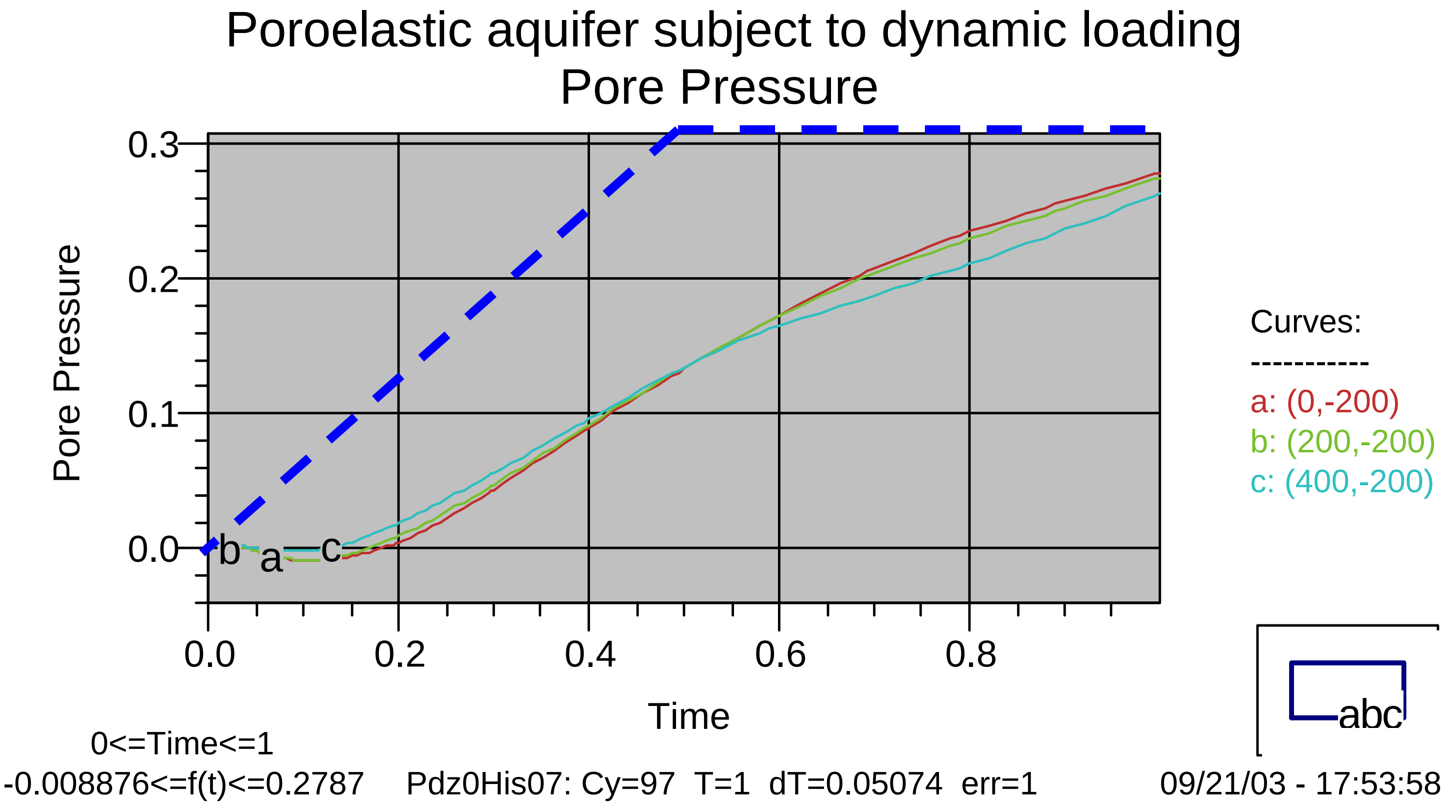
$$f = \begin{cases} t \times 10^5 \text{ N} & \text{for } t \le 0.1\,\text{s} \\ 10^4 & \text{for } t > 0.1\,\text{s} \end{cases}$$

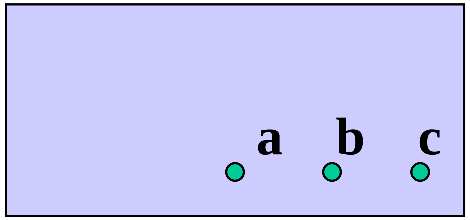


# A "flat" ramp:

$$f = \begin{cases} 2t \times 10^4 \text{ N} & \text{for t} \le 0.5\,\text{s} \\ 10^4 \text{ N} & \text{for t} > 0.5\,\text{s} \end{cases}$$



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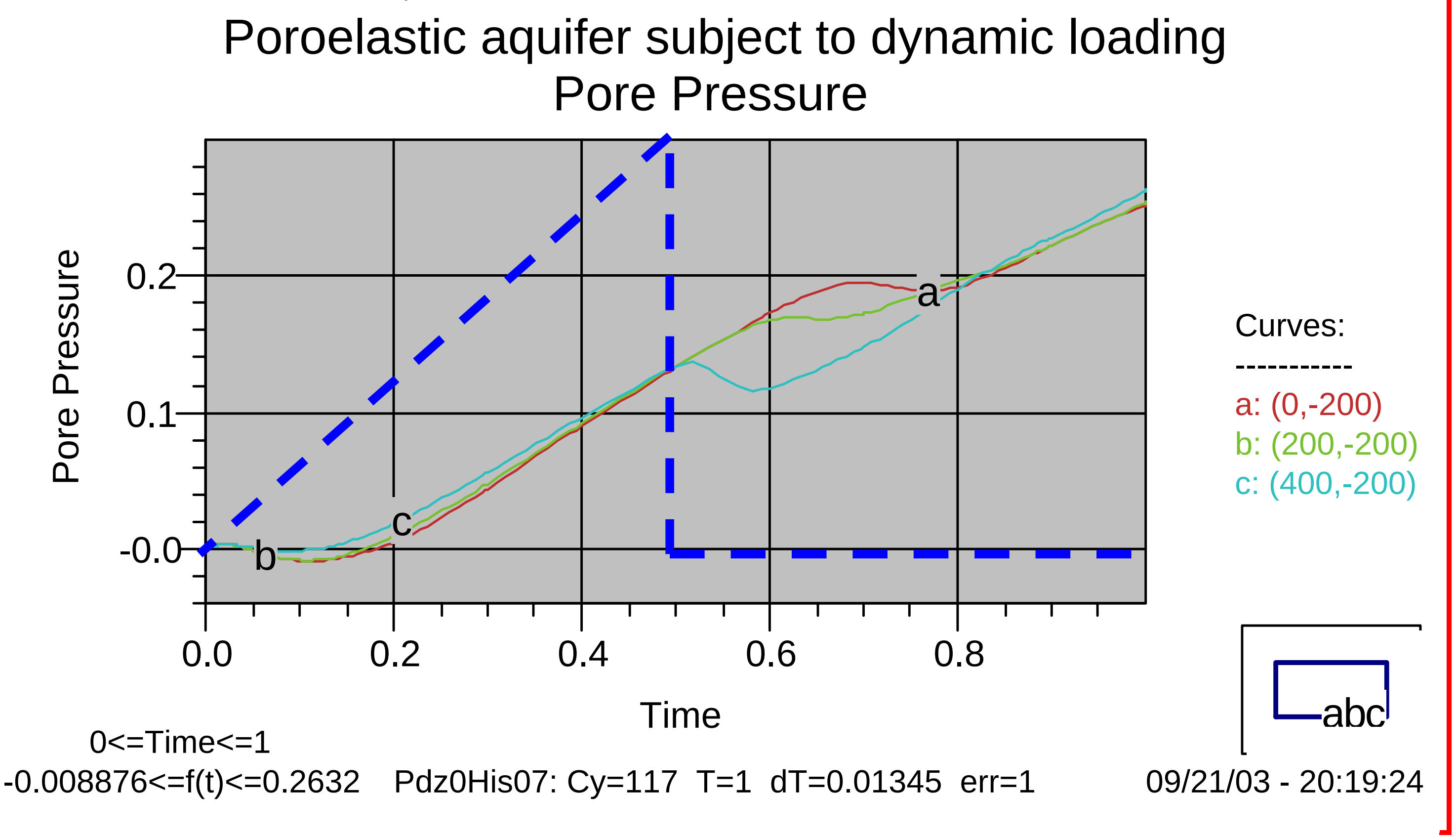
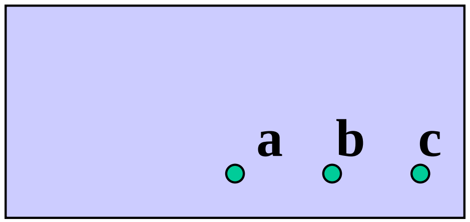
# A “flat” ramp then drop:

$$f = \begin{cases} 2t \times 10^4 \ \text{N} & \text{for } t \le 0.5\,\text{s} \\ 0 \quad \text{N} & \text{for } t > 0.5\,\text{s} \end{cases}$$





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# Still a long way to go . . .

- Sensitivity studies:
  - Every parameters in the model should be evaluated.
- Does this model make sense?
- How to fit this model into the field data?
- Make a more complex model:
  - Non-homogeneity
  - Anisotropy
  - Discontinuity
  - Non-linearity
  - Real time quake simulation


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資源工程學系
Department of Resources Engineering

# Doggerel

- This report does not shed any light,
  not because the budget is tight,
  but because I really have nothing to hide.
- Prediction? Not quite. Animation? Aye.
  Using appropriate software guide,
  modeling is a delight, not plight.
- When in doubt, simple mind always makes you feel right.


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