Studies on the Elastic Wave Velocity in Clastic Rock

By

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Part I. Effect of Water Content on Elastic Wave Velocity in Clastic Rock

1. Introduction

The effect of water content on the elastic wave velocity in clastic rock has been studied by many investigators : K. Iida[4)5)6)], D. S. Hughes[3)], J. E. White, R. L. Sengbush[15)] and others.

Some discrepancy in their results, however, shows us that the effect should be examined and explained in connection with texture, degree of lithification and physical property in clastic rock.

The examination of the effect on the bases of the physical property of clastic rock will lead us to a better understanding of velocity controlling factors, and undoubtedly promote the geological interpretation of seismic data.

The writer carried out some experiments on medium and coarse sandstone, whose elastic wave velocity ranges from about 2 km/sec to 4 km/sec, and on a few samples of shale.

2. Experimental procedure

Longitudinal wave velocity was measured by using ultra-sonic pulse method, which was pioneered by D. S. Hughes[2)] and developed by many others[7)10)12)]. The principle of our measuring apparatus is similar to those of the above investigators. A brief description of the block diagram and circuit diagram is presented in the Appendix I. $BaTiO_3$ disc of 30 mm in diameter and 4 mm in thickness were used for transmitter and receiver respectively. An example of photographic records is illustrated in Fig. 1.

The size of a rock specimen is about 5 cm in diameter and 5～8 cm long. In order to achieve water saturation, the dry specimen was placed in a metal box attached to a vacuum pump, which was run 1 hour at 0.002 atm. and then was immersed in distilled water which had been de-aerated in vacuum. Air was then slowly admitted, and the specimen was kept in water for 7 days.

Velocity variation was measured in the evaporation process at room temperature (about 25° C) and atmospheric pressure. After natural evaporation, specimen was dried by using infra-red ray, and then in an oven at 110° C for 2 hours.

3. Experimental results

The samples used in this experiments were described in Table 1.

Variation of longitudinal wave velocity with respect to water content in medium and coarse sandstone is illustrated in Fig. 2, and that in shale in Fig. 5 and Table 2. Relation between velocity and porosity in sandstone is illustrated in Fig. 4. In sandstone, the wave velocity-water content curve is generally composed of 2 components, namely, one increasing due to "pendicular stage" and the other decreasing due to the "funicular stage" which are discussed under **4**.

4. Discussion of results

Water in clastic rock is classified into crystalline, adsorbed, absorbed, contact moisture and free water according to its condition. The condition where free water exist is generally called "funicular stage", and the condition where contact moisture exist is called "pendicular stage". Among these waters, only free water and contact moisture are thought to affect the elastic property of clastic rock under the present experimental condition.

The process through which each water affects the elastic property may be classified into (1) mechanical process, which is the mechanical interaction between water and elastic constituent particles, and (2) chemical process, which involves complex chemical interaction between water and cementing materials. The latter process, as shown in the textbook of soil mechanics[9)13)], is so complex that the following discussion is limited to the former process. The mechanical effect of each water is examined by several models, F. Gassmann[1)] and T. Takahashi[14)].

1) Pendicular Stage

In the pendicular stage, capillary pressure results from contact moisture in the granular substance.

The capillary pressure was calculated for various particle sizes and tabulated in Table 3.

As regards constituent particles, the capillary pressure is equivalent to the external pressure. The effect of external pressure on the elastic property of granular substance has already been treated by T. Takahashi and Y. Satō[14]. The model, however, did not explain the high velocity in clastic rock under the atmospheric pressure. A modification was made on their theory to the case where the particles are bound at the contact plane. The description of the theory is presented in Appendix II.

The longitudinal wave velocity $V_p{}^*$ through the granular substance, where the particles are bound at the contact plane, is

$$V_p{}^* = \sqrt{V_1{}^2 + V_0{}^2}$$

where

$$V_1{}^2 = 0.865\left(\frac{n}{10}\right)^{2/3}\left(\frac{\rho^*}{\rho}\right)^{-2/9}\left(\frac{1}{\rho}\right)^{1/3} V_p{}^{4/3} P^{1/3}.$$

$P = P_{01} + P_{02}$

V_0 : longitudinal wave velocity when $P=0$

V_p : longitudinal wave velocity in the particle

P_{01} : external pressure

P_{02} : capillary pressure

n : number of contact sphere

ρ : density

ρ^* : apparent density of the granular substance

Variation of the velocity with respect to the pressure is graphically illustrated in Fig. 1 of Appendix II, and is numerically tabulated in Table 4 for the pressure range 0–100 bar. By using Tables 3 and 4, velocity increase due to capillary pressure can be found for both the various sizes and various velocity V_0. For example, when the velocity V_0 is 2.0～4.0 km/sec, velocity increase is only 30 m/s～10 m/s in the case of particle size of 0.1 mm, and is 200m/s～100m/s in the case of particle size of 1×10^{-5} mm. Consequently, the velocity increase due to drying in sandstone, as seen in the left side of Fig. 2, is due to capillary pressure in the matrix and cement.

No velocity increase was found in shale as it dried. The granular model of clay size does not fit for shale. This will be due to the property of shale which is likely to break into pieces with water saturation. Dr. Iida's experiment on soft sediment did show velocity increase. This suggest the further study on the change of elastic property through lithification from soft sediment to shale.

2) Funicular Stage

In funicular stage, the free water play a role in the elastic property of rock through mechanical interaction with the constituent elastic particles. F. Gassmann[1] studied the effect of saturated water on the elastic property of porous medium. According to his paper, the relations of elastic constants between saturated and dry conditions are

$$k = \hat{k}\,\frac{\bar{k}+Q}{\hat{k}+Q}, \quad Q = \frac{\tilde{k}(\hat{k}-\bar{k})}{n(\hat{k}-\tilde{k})}$$

$$\mu = \bar{\mu},$$

$$\rho = \bar{\rho} + n$$

$$\bar{\rho} = \hat{\rho}(1-n),$$

where

k, $\bar{k}$: bulk modulus of porous media, in saturated and dry, respectively

μ, $\bar{\mu}$: rigidity modulus of porous media, in saturated and dry, respectively

ρ, $\bar{\rho}$: apparent density of porous media, in saturated and dry, respectively

$\hat{k}$, $\tilde{k}$: bulk modulus of constituent material and water

$\hat{\rho}$: density of constituent material

n : porosity

The above relation between bulk modulus k and $\bar{k}$ is graphically illustrated in Fig. 7. F. Gassmann assumed that rigidity of the framework does not change with water saturation. Instead of this assumption, we shall be able to assume that Poisson's ratio does not change with water saturation.

Under each assumption, wave velocity variation in water saturated condition was calculated, and is illustrated in Fig. 7 (in the case of $\sigma=\bar{\sigma}$) and in Fig. 8 (in the case of $\mu=\bar{\mu}$). Comparison of experimental data and these calculated values is tabulated in Table 5.

In this comparison, minimum velocity in the experiments corresponds to the velocity in the dry condition in F. Gassmann's theory, because the velocity in dry condition in his theory means the velocity in the framework. For sandstone, the calculated velocity increase in the saturated condition is about 50 % of the experimental results, assuming $\mu=\bar{\mu}$. However, assuming $\sigma=\bar{\sigma}=0.25$, the agreement between the calculated and the experimental value is very good.

The variation of Poisson's ratio with saturation will be tested in the future.

5. Summary and conclusion

Variation of longitudinal wave velocity with water content was measured for medium and coarse sandstone and shale specimens, which were obtained from sample cores of Paleogene coal bearing formation. The experimental results were examined by two models.

The factors which control the elastic property of rock with water saturation at room temperature and atmospheric pressure are (1) the elastic property of the framework itself, (2) internal pressure caused by capillary pressure in the matrix and cement, and (3) the mechanical interaction between elastic particles and free water.

The wave velocity—water content curve has two components, one increasing and the other decreasing with water content. The former is the effect of the mechanical interaction between the elastic particles and free water, and the latter is the effect of internal pressure in the matrix and cement.

Results for sandstone are as follows :

(1) The longitudinal wave velocity decreases greatly as saturation reduced from 100% to 20～30%. The velocity variation with saturation is controlled by the elastic property of framework and the interaction between the elastic particles and free water. The amount of this velocity variation in the saturated condition is explained very well by F. Gassmann's theory, if his assumption of rigidity invariant ($\mu=\bar{\mu}$) with saturation is replaced by the new assumption of Poisson's ratio invariant ($\sigma=\bar{\sigma}$).

(2) As the water content is reduced from 20～30 % to the dry condition, the longitudinal wave velocity increases again. This velocity variation is controlled by the internal pressure and the framework.

The amount of this velocity increase with drying is explained by capillary pressure in the matrix and cement in sandstone.

The result for shale is as follows :

(3) The wave velocity is nearly constant from 100 % to 30～40 % saturation and then decreases. Any velocity increase in a dry condition is too small to be measured.

The model for granular substances of clay size does not seem suitable to shale.

Appendix I. Apparatus

A brief description of the apparatus is given with the block diagram (Fig. 1) and the circuit diagrams (Figs. 2, 3, 4).

Appendix II. On the elastic property of granular substances where particles are bound at the contact plane

T. Takahashi and Y. Satō's theory[14] on the elastic property of granular substances is extended to the case where the particles are bound at the contact plane. Binding force between particles, which were taken into consideration, are

(1) binding force due to the external pressure,

(2) binding force due to the internal pressure caused by contact moisture,

(3) binding force due to cementing material.

Elastic wave velocity through dry clastic rock under atmospheric pressure will depend upon the latter two factors.

Elastic energy per unit volume is calculated by static stress-strain consideration. The elastic wave velocity is, then, obtained by using Takahashi—Satō's theory.

The longitudinal wave velocity V_p^* through the granular substance, where particles are bound at the contact plane, becomes

$$V_p^*=\sqrt{V_1^2+V_0^2}$$

where

$$V_1^2=0.865\left(\frac{n}{10}\right)^{2/3}\left(\frac{\rho^*}{\rho}\right)^{-2/9}\left(\frac{1}{\rho}\right)^{1/3}V_p^{4/3}P^{1/3}$$

$P \ =P_{01}+P_{02}$

V_0 : longitudinal wave velocity when $P=0$

V_p : longitudinal wave velocity of material of particle

P_{01} : external pressure

P_{02} : capillary pressure

n : number of contacting spheres

ρ : density of material of the particle

ρ^* : apparent density of the granular substance

Part II. Progressive Change of Elastic Property through Lithification in Sandstone

1. Introduction

As stated in Part I, information on the elastic properties such as the framework, porosity and internal pressure in sandy clastic rock can be obtained by analyzing the variation of longitudinal wave velocity with water content. The writer investigated the progressive change of elastic property through lithification in sandstone by applying this method to various sandy clastic rock.

As for the young sediment, the experiment was made on the soft siltstone in Pleistocene and the elastic property of soft sediment is examined in **2**.

Development of framework through lithification is studied with respect to the porosity in **3**, by separating the elastic wave velocity in the framework itself from the water content—velocity curve. From these data, progressive change of elastic property through lithification is examined in **4**. Based on these results, the writer discussed in **5** some problems on seismic prospecting concerning the method of the survey technique and that of interpretation.

2. Variation of elastic wave velocity with water content in siltstone

The core used in this experiment was taken from the sample cores in the Pleistocene formation of the Ishikari coal field. The density in water saturation is 1.81; the effective porosity is 47.3%; depth of sampling is 322 m.

The experiment was carried out by the same procedure as that of Part I, and the variation of longitudinal wave velocity with respect to water content is illustrated in Fig. 1. The curve for this high porous siltstone is very similar to the curves which were obtained in indurated sandstone.

Discussion of results :

(i) Velocity increase due to drying

Comparing the observed velocity increase due to drying with the calculated value (Table 1), which method is described in Part I, Appendix II, the velocity increase is thought to be caused by internal pressure in the matrix and cement.

(ii) Velocity in water saturated state

Comparing the observed velocity variation in a water saturated state with calculated value for various models, namely for F. Gassmann's porous model of $\sigma=\bar{\sigma}$ and $\mu=\bar{\mu}$, and for the mixture model, the observed velocity variation is quantitatively explained very well by F. Gassmann's model of $\mu=\bar{\mu}$ (Table 2). The assumption of Gassmann's model of $\sigma=\bar{\sigma}$ means that the rigidity μ of the porous medium increases in a water saturated state over that of the framework as the bulk modulus k increases. While the assumption of Gassmann's model of $\mu=\bar{\mu}$ means that rigidity μ remains constant in a water saturated state in spite of increasing of bulk modulus. The variation of Poisson's ratio of the soft siltstone with water content is examined by taking into consideration the physical meaning of each assumption.

Poisson's ratio σ is a function of μ/k, $\sigma=(3-2\mu/k)/(6+2\mu/k)$, and the relation is illustrated in Fig. 2. Poisson's ratio of the framework of soft siltstone is about 0.2~0.3 and $\bar{\mu}/\bar{k}=0.6(\bar{\sigma}=0.25)$, according to Dr. Iida's experiment (1940). The bulk modulus of the framework $\bar{k}$ is rather small. In water saturated state, however, the bulk modulus k of the rock increases because of interaction between particles and water. Therefore, $\bar{k}/k$ becomes very small. On the otherhand, rigidity μ remains constant under the assumption of $\mu=\bar{\mu}$. Therefore, μ/k in a saturated state becomes very small with the result that Poisson's ratio approaches 0.5. Comparison of the calculated values of mixture model with the observed values will agree with the above interpretation.

From these considerations, we conclude for the elasticity of high porous siltstone that (1)

F. Gassmann's porous model is suitable, (2) the rigidity in water saturated state is nearly the same as that of the framework, (3) Poisson's ratio in the water saturated state approach 0.5, while that of the framework is 0.2～0.3.

3. Development of the framework with respect to porosity

Because the variation of longitudinal wave velocity with water content is quantitatively explained very well by F. Gassmann's porous model, the elastic property of the framework can be separated from the velocity—water content curve; the minimum velocity on the curve is due to the elasticity of the framework. In order to investigate the development of the framework due to lithification, the longitudinal wave velocity in the framework was measured for many sandstone samples, which are given in Table 3. The longitudinal wave velocity in the framework with respect to porosity is illustrated in Fig. 3.

From this graph, we notice as a general tendency that the velocity in the framework becomes large as the porosity becomes small. From the precise view, however, the variation of the velocity in the framework at the same porosity is very wide especially when porosity becomes less 20%. The general tendency between velocity in the framework and porosity will be due to compaction and cementation. The wide variation in the less porous ranges, however, is due to both the texture of the rock and the quality of the cementing materials.

4. Progressive change of elastic property through lithification in sandstone

The longitudinal wave velocities in the saturated state and the framework are plotted with respect to porosity in Fig. 4. Curve (1) in the Fig. 4 shows the calculated velocity in the mixture model of elastic particles and water. Curve (2) shows the calculated velocity of the assumed mixture model when $\sigma \doteqdot 0.25$. The observed longitudinal wave velocity in the saturated condition is a little larger than curve (1) in the high porous region. As porosity decreases (50～30%), the observed velocity deviates upwards from curve (1). When porosity is less than about 15%, the velocity becomes larger than curve (2).

From curves and experimental data described in this report, progressive changes of the elastic property through lithification in sandstone is examined and schematically summarized in Table 4. For the very young sediment with porosity ranges about 100～50%, the elastic property is approximately determined by the mixture model of elastic particles and water.

The framework begins to develop in the porosity range of about 50～20%, but is still weak. Therefore, contribution of the framework in a saturated state to the bulk modulus is small; the main contribution coming from the mixture model; the rigidity remains nearly constant as that of the framework; and Poisson's ratio approach about 0.5. When lithification develops very much and porosity decreases less than 20%, the elastic property of the framework becomes very strong. In a water saturated state, both the bulk modulus and rigidity become large so that Poisson's ratio does not change with water content.

5. Remarks on seismic prospecting

The following remarks on seismic prospecting are derived from the above results.

(1) Elastic property of low velocity surface layer

By considering the elastic property of low velocity surface layer, particularly concerning the rigidity, it was remarked that many field techniques which aim at the reduction of surface ground roll should be controlled by the shear wave velocity distribution in the formation.

(2) On the reflection bed

From the physical view point, the reflection horizon is merely a boundary of acoustic impedance. The elastic wave velocity is mainly controlled by porosity and the framework. These factors have close relation with the classification and description of geological lithofacies. Therefore, the reflection bed should be correlated first to the lithofacies.

(3) On the correlation between seismic unit and stratigraphic units

Correlation procedure is schematically presented in Fig. 5. Correlation of velocity layer and reflection, refraction horizon to the stratigraphic unit should be studied with the stratigraphic analysis such as sand-shale ratio, iso-pack and other analysis.

6. Summary and conclusion

Progressive change of elastic property in sandy clastic rock was studied by carrying out experiments on the variation of the longitudinal wave velocity with water content.

As regards the elastic property of soft siltstone (porosity : 47.3%; the longitudinal wave velocity in the water saturation state : 1.54 km/sec) in a Pleistocene formation, results obtained are;

(1) The longitudinal wave velocity variation with water content is controlled by the

elastic property of framework, porosity and internal pressure. F. Gassmann's model of $\mu=\bar{\mu}$ seems to be suitable to such high porous siltstone.

(2) The longitudinal wave velocity in a saturated state mainly depends upon the mixture model of elastic particles and the water, and the velocity in a partially saturated state decreases to the velocity in the framework.

(3) The shear wave velocity is determined by the elasticity of the framework, and the rigidity does not change with water content.

(4) Poisson's ratio in the saturated state is a little less0. 50.

As regards the development of the framework through lithification, results are;

(1) As a general tendency, the longitudinal wave velocity in the framework increases as the porosity decreases.

(2) When the porosity becomes less than about 15%, there seems to be no close relation between porosity and the velocity in the framework.

Progressive changes of elastic properties in sandy clastic rock are summarized in Table 4.

Remarks on seismic prospecting are;

(1) Shear wave velocity should be carefully examined for controlling the surface waves.

(2) Reflection bed should be correlated first to the lithofacies.

(3) A correlation procedure between seismic units and stratigraphic units is schematically illustrated in Fig. 5.